AQ061-3-M-ODL-TSF Time Series Analysis and Forecasting

Topic 2 – Smoothing Techniques (Part I)

TOPIC LEARNING OUTCOMES

At the end of this topic, you should be able to:

1. have a broad understanding of forecasting techniques, what the most commonly used methods are and how to integrate them into decision making process.
2. select an appropriate model for time-related data; learn what the methods can and can't do, what their strengths and weaknesses are; analyse the data, with or without software and interpret the result.
3. solve the model using computer software and interpret the results.

Contents & Structure

- Moving average, Weighted / Exponential and Modified Moving average
- Exponential Smoothing
- Holts Method
- Linear / Quadratic / Exponential Trend
- Decomposition Model
- Holts Winter Smoothing
- Dummy variables + Regression

Recap From Last Lesson

- Questions to ask to trigger last week's key learning points

Forecasting

Predict the next number in the pattern:

a) 3.7, 3.7, 3.7, 3.7, 3.7,

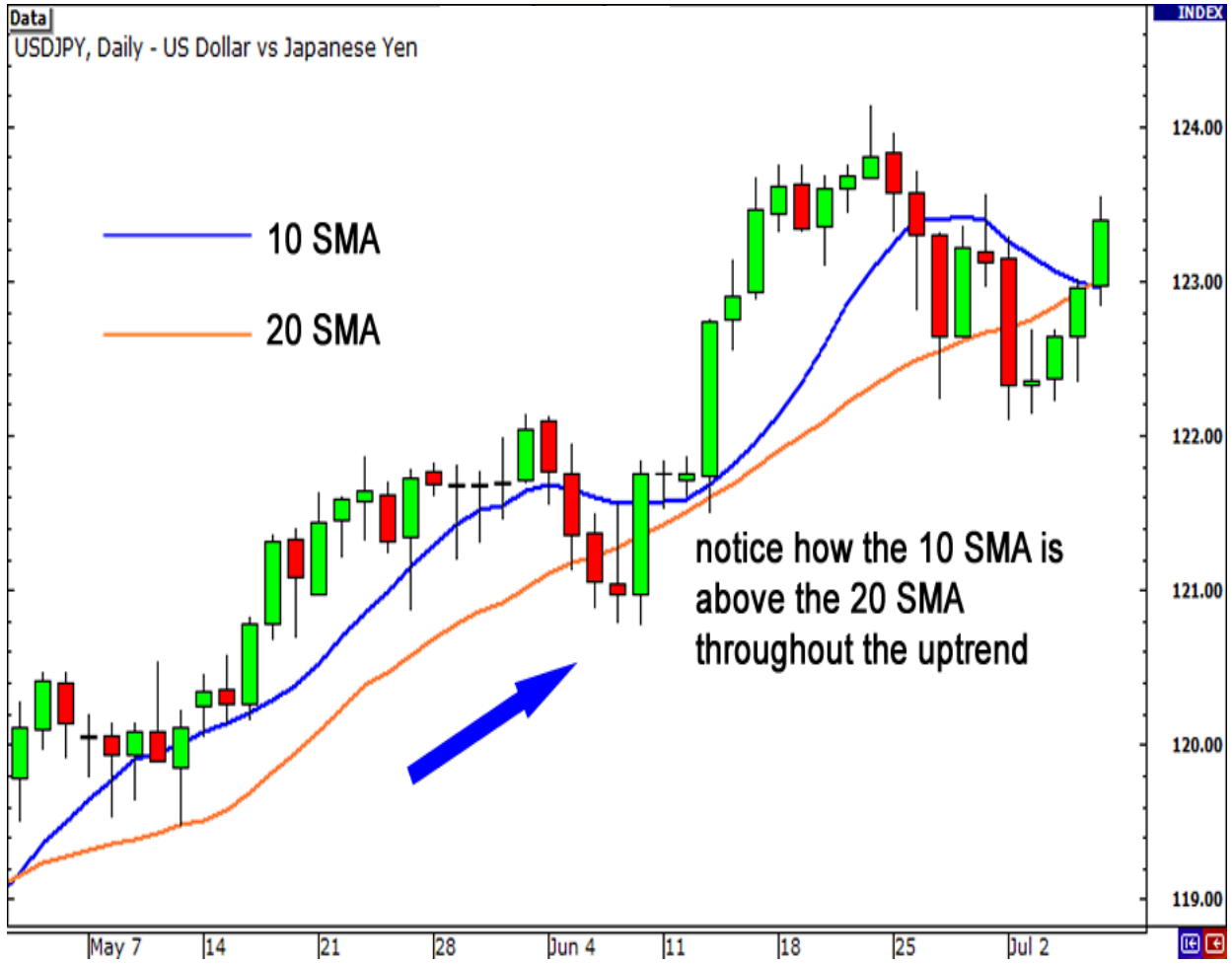
b) 2.5, 4.5, 6.5, 8.5, 10.5,

c) 5.0, 7.5, 6.0, 4.5, 7.0, 9.5, 8.0, 6.5,

Moving Average

- Uses the mean of all relevant historical observations as the forecast of the next period – does not display any pronounced behavior, such as trend or seasonal pattern.
- average of last few actual data values, updated each period.
  - First t-data as the initialization part and remaining data as a test part.
  - Average the initialization part of the data and to forecast the next period.
  - When new observation becomes available, forecast for the next is updated by adding the newest value and dropping the oldest and recalculating the average.

Example



Random fluctuations
provide no help in forecasting

Aspects: Smooth away the rapid fluctuations, and recent behaviour is often a good indication in the near future.

Method: Averaging adjacent values in the series – mean (vary less than original data)

Pattern: No regular repeating pattern (seasonal effect).

Exercise

Develop **3-week Moving Average** forecasts for the below demand.

Week	Demand
1	650
2	678
3	720
4	785
5	859
6	920
7	850
8	758
9	892
10	920
11	789
12	844

Moving Average

- Useful if we can assume that market demands will stay fairly steady over time.
- Tend to smooth out the variability occurring in the actual data.
- Long-period MA average smooth out fluctuation to a greater extent than shorter-period MA.
- However, it reacts more slowly to recent demand changes than do shorter-period MA.
- The choice of the interval for the moving average depends on the length of the underlying cycle or pattern in the original data.

Moving Average

- The stability of the demand series generally determines how many periods to include
  - A small number is most desirable when there are sudden shifts in the level of the series.
  - A large number is desirable when there are wide, infrequent fluctuations in the series.

Weighted Moving Average

- Weights are assigned to most recent data to show its emphasis - more responsive to changes because more recent periods maybe more heavily weighted.
- choice of weights is somewhat arbitrary
  - Determine the precise weights to use require some trial and error.
  - If the most recent data are weighted too heavily, the forecast might overreact to a random fluctuation in orders.
  - If they are weighted too lightly, the forecast might under react to an actual change in the pattern of orders.

$$WMA = \frac{\sum (\text{weight for period n})(\text{demand in period n})}{\sum \text{weights}}$$

Exercise

Develop **3-week Weighted Moving Average** forecasts for the below demand, with weights = 0.7, 0.2 and 0.1 to the most recent, next recent and the most distant data.

Week	Demand
1	650
2	678
3	720
4	785
5	859
6	920
7	850
8	758
9	892
10	920
11	789
12	844

Variations of Moving Average

- Exponential Moving Average

$$EMA_i = EMA_{i-1} + \alpha(x_i - EMA_{i-1})$$

$$\alpha = \frac{2}{n+1}$$

- Modified Moving Average

$$MMA_i = MMA_{i-1} + \frac{1}{n}(x_i - MMA_{i-1})$$

Exercise

Develop **3-week exponential** and **modified Moving Average** forecasts for the below demand.

Week	Demand
1	650
2	678
3	720
4	785
5	859
6	920
7	850
8	758
9	892
10	920
11	789
12	844

Exponential Smoothing

- A weighted moving average forecasting technique in which data points are weighted by an exponential function
  - The weights used are α for the most recent observation, $\alpha(1-\alpha)$ for the next most recent and $\alpha(1-\alpha)^2$ for the next and so forth.
- An averaging methods that weights the most recent past data more strongly than most distant past data
  - Is a procedure for continually revising a forecast in the light of more recent experience

Exponential Smoothing

- The forecast will react more strongly to immediate changes in the data.
  - This is very useful if the recent changes in the data are the result of an actual changes instead of just random fluctuation.
- Often appropriate for data with no predictable upward or downward trend.
  - The aim is to estimate the current level which is then used as the forecast of future values.

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

or

$$F_{t+1} = \alpha A_t + (1 - \alpha)F_t$$
$$= \alpha A_t + \alpha(1 - \alpha)A_{t-1} + (1 - \alpha)^2 F_{t-2} + \ldots$$

Example

Determine exponential smoothing forecasts for periods 2-10 using α **= 0.1** and α **= 0.6**, assuming $F_1 = D_1$

Week	Demand
1	650
2	678
3	720
4	785
5	859
6	920
7	850
8	758
9	892
10	920
11	789
12	844

Exponential Smoothing

- Forecasts are computed for α equal to .1, .2, … .9 and the sum of the squared forecast errors is computed for each. The value of α producing the smallest error is chosen for use in generating future forecasts.
- When demand is relatively stable, a small value for α is more appropriate.
- When actual demand displays an increasing trend, a larger value of α is generally better.
- Select α close to 1 if you wish the forecast values to depend strongly on recent changes in the actual values.

Exercise

Table below shows the number of umbrellas sold at a clothing store over the past 3 years.

Year	Quarter	Sales	Year	Quarter	Sales
1	1	125	3	1	138
	2	153		2	144
	3	106		3	113
	4	88		4	80
2	1	118			
	2	161			
	3	133			
	4	102			

Use 4-period simple Moving Average, Exponential and Modified Moving Average to forecast the sales.

Practical Exercise

You are required to analyse the data below using Moving Average and Simple Exponential Smoothing:

quakes.dat – annual number of earthquakes worldwide with a magnitude greater than 7.0 on the seismic scale

Double Exponential Smoothing

- Also known as **Holt Method**.
- Used when observed data is trending and contain information that allows the anticipation of future upward movements.
- Adjusted exponential smoothing is the exponential smoothing forecast with an adjustment for a trend added to it.
- Holt's technique smoothes the level and slope directly by using different smoothing constants for each.
- Large weights result in more rapid changes in the component; small weights result in less rapid changes.

Double Exponential Smoothing

$$F_{t+k} = L_t + kT_t$$

where:

$L_t = \alpha A_t + (1 - \alpha)(L_{t-1} + T_{t-1})$

$T_t = \beta (L_t - L_{t-1}) + (1 - \beta)T_{t-1}$

A_t = actual value in period t

β = smoothing constant for trend ($0 \leqslant \beta \leqslant 1$).

α= smoothing constant for data ($0 < \alpha < 1$).

k = the number of periods ahead to be forecast

Example

Determine Holts method (double exponential smoothing) forecasts for periods 2-10 using α **=0.6** and β **=0.3**, assuming $L_1 = D_1$ and $T_1 = D_2 - D_1$

Week	Demand
1	650
2	678
3	720
4	785
5	859
6	920
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Exercise

A manufacturer wants to forecast the demand for a pollution-control equipment.

To forecast the demand on October, use

(i) **Linear trend model**

(ii) **Holts method** assumes the initial level for month 1 was 12 units and the trend over that period was 0. Using $\alpha = 0.2$ and $\beta = 0.4$

Month (t)	Demand (Y_t)
1	12
2	17
3	20
4	19
5	24
6	21
7	31
8	28
9	36
10	

Other Models to Consider

- Linear Trend Model

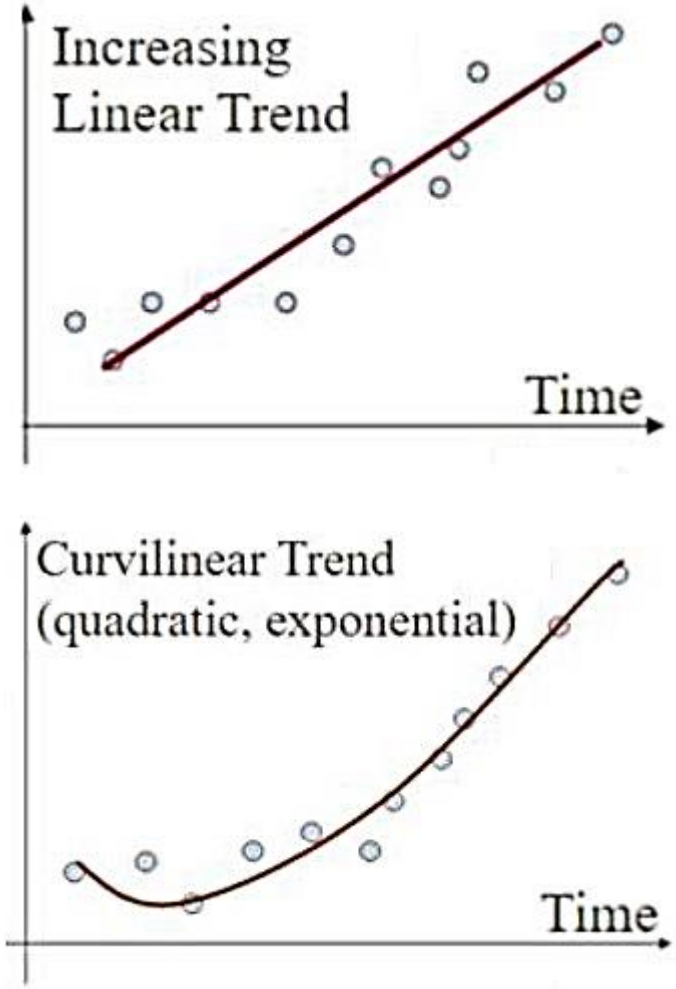
$$F_t = a + bt$$

- Quadratic Trend Model

$$F_t = a + bt + ct^2$$

- Exponential Trend Model

$$F_t = a.\exp(bt)$$



Practical Exercise

Perform time series analysis for the data below. You are required to use **THREE** of the following forecasting techniques to perform the forecast:

- **Moving average or Weighted Moving Average**
- **Modified Moving Average or Exponential Moving Average**
- **Simple Exponential Smoothing**
- **Linear, Quadratic or Exponential Trend Model**
- **Holts Method**

Population data in Malaysia from 1970 to 2019. You may download the data by clicking the link given:
https://www.dosm.gov.my/v1/index.php?r=column/ctimeseries&menu_id=bnk3bk0wTTkxOXVHaVg3SUFDMlBUUT09

Tahun.Year	Jumlah.Total	Age0_14	Age15_64	Age65+	Average_Growth
1	1970	10881.8	4847.3	5677.6	356.8	3.6
2	1971	11159.7	4907.0	5876.4	376.3	2.5
3	1972	11441.3	4975.1	6076.9	389.3	2.5

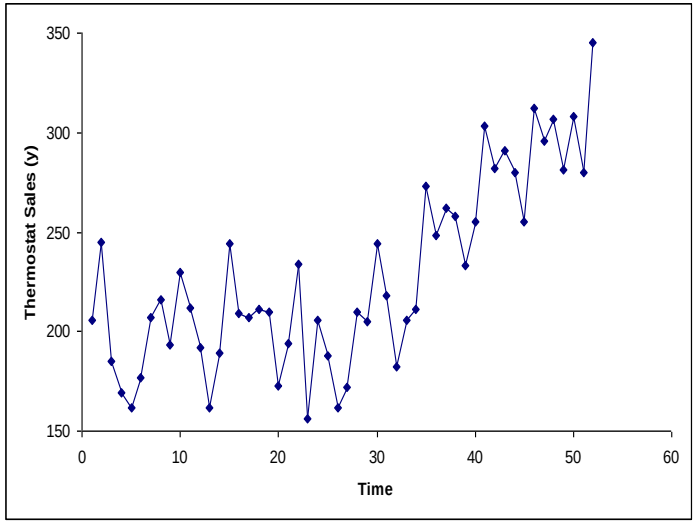
Example

- Use the example of Thermostat Sales as an illustration

- Findings:
  - Overall an upward trend
  - The growth rate has been changing over the 52-week period
  - There is no seasonal pattern

  ⇒ Holt's exponential smoothing method can be applied

Weekly Thermostat Sales			
206	189	172	255
245	244	210	303
185	209	205	282
169	207	244	291
162	211	218	280
177	210	182	255
207	173	206	312
216	194	211	296
193	234	273	307
230	156	248	281
212	206	262	308
192	188	258	280
162	162	233	345



Review Questions

Summary / Recap of Main Points

- understand of forecasting techniques, what the most commonly used methods are and how to integrate them into decision making process.
- select an appropriate model for time-related data; learn what the methods can and can’t do, what their strengths and weaknesses are; analyse the data, with or without software and interpret the result.
- solve the model using computer software and interpret the results.

What To Expect Next Week

In Class

Preparation for Class

Performance Evaluation