AQ061-3-M-ODL-TSF Time Series Analysis and Forecasting

Topic 2 – Smoothing Techniques (Part II)

TOPIC LEARNING OUTCOMES

At the end of this topic, you should be able to:

1. have a broad understanding of forecasting techniques, what the most commonly used methods are and how to integrate them into decision making process.
2. select an appropriate model for time-related data; learn what the methods can and can't do, what their strengths and weaknesses are; analyse the data, with or without software and interpret the result.
3. solve the model using computer software and interpret the results.

Contents & Structure

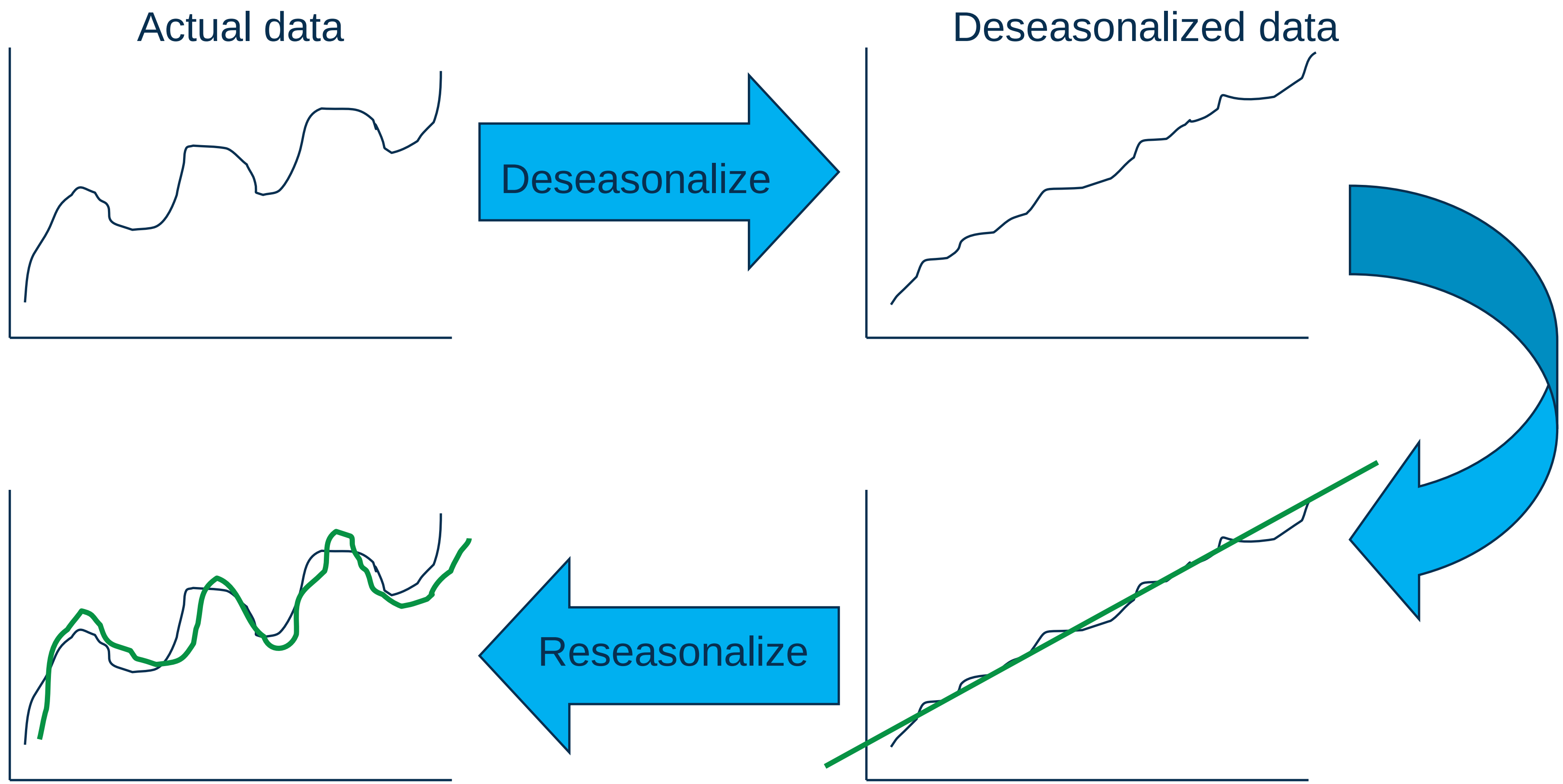
- Moving average, Weighted / Exponential and Modified Moving average
- Exponential Smoothing
- Holts Method
- Linear / Quadratic / Exponential Trend
- Decomposition Model
- Holts Winter Smoothing
- Dummy variables + Regression

Recap From Last Lesson

- Questions to ask to trigger last week’s key learning points

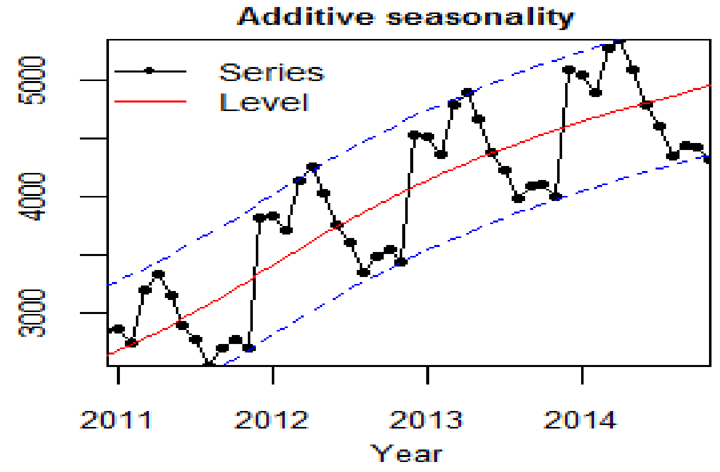
Decomposition Model

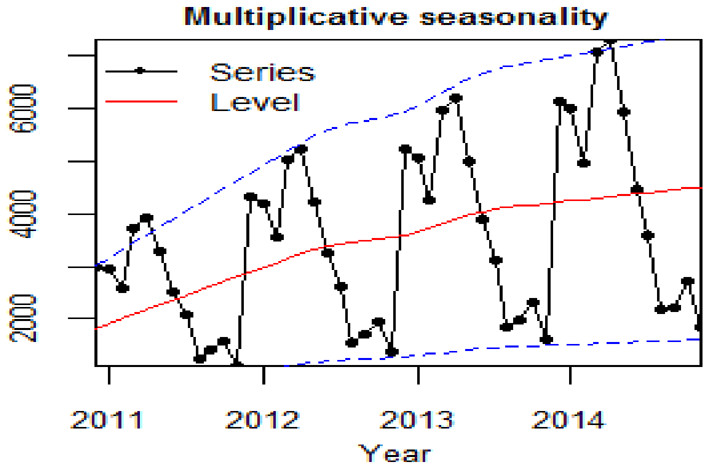
- **Example (SV + Trend)**



Basic Models of Decomposition Model

- Basic structures:
  - Additive, Y : trend (T) + seasonal (S) + random (I)
  - Multiplicative, Y : trend (T) × seasonal (S) × random (I)
- Additive model is useful when the seasonal movements are the approximately same from year to year.
- Multiplicative model is useful when the amplitude of seasonality increases (decreases) with an increasing (decreasing) trend.

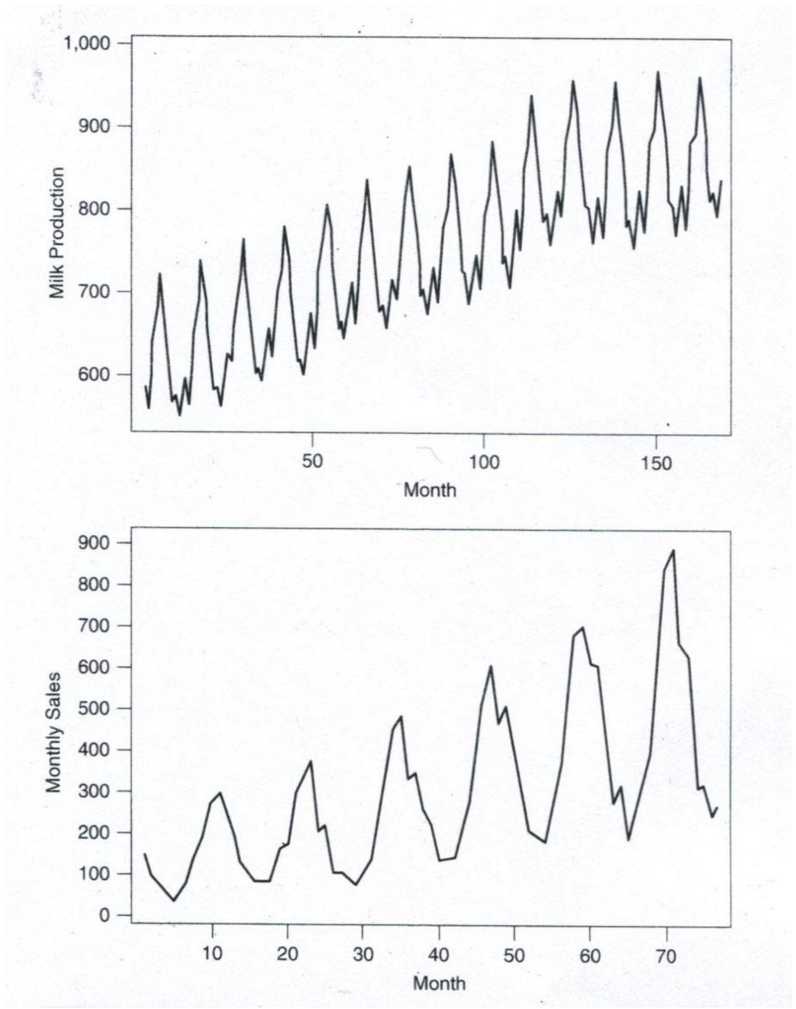




Basic Models of Decomposition Model

a) A time series with constant variability

b) A time series with variability increasing with trend



Additive Decomposition Model

- It is used when
  - The deviation from the trend is fairly stable (i.e: the seasonal movements are the approximately same from year to year).
  - The behaviors of the component are independent from each other, vary around 0(i.e: trend-cycle will not cause an increase in the magnitude of seasonality).
- The original time series is a sum of its components, so the seasonally adjusted data are calculated as:

$$SA_t = Y_t - S_t$$

- Seasonally adjustment allows reliable comparison of values at different points in time – seasonal variation is not of primary interest.

Additive Decomposition Model

- Techniques For Calculating Seasonal variation
  - Use Moving Average (CMA) to calculate the trend (t) values (remove seasonal component, S).
  - Calculate for each time point, the value of y – t (the difference between the original value and the trend)
  - For each seasonal in turn, find the average of all the y – t values (remove irregular component, I).
  - If the total of the averages differs from zero, adjust one or more of them so that their total is zero. (average to zero , sum of seasonal variation is small, so the seasonal effect within a year cancels itself out)

Additive Decomposition Model

- Technique for forecasting

$$Y_{est} = T_{est} + \text{adjusted seasonal variation}$$

where:

Y_{est} = estimated data value

T_{est} = projected trend value

Example (Additive Decomposition Model)

The following set of data shows the UK outward passenger movements by sea.

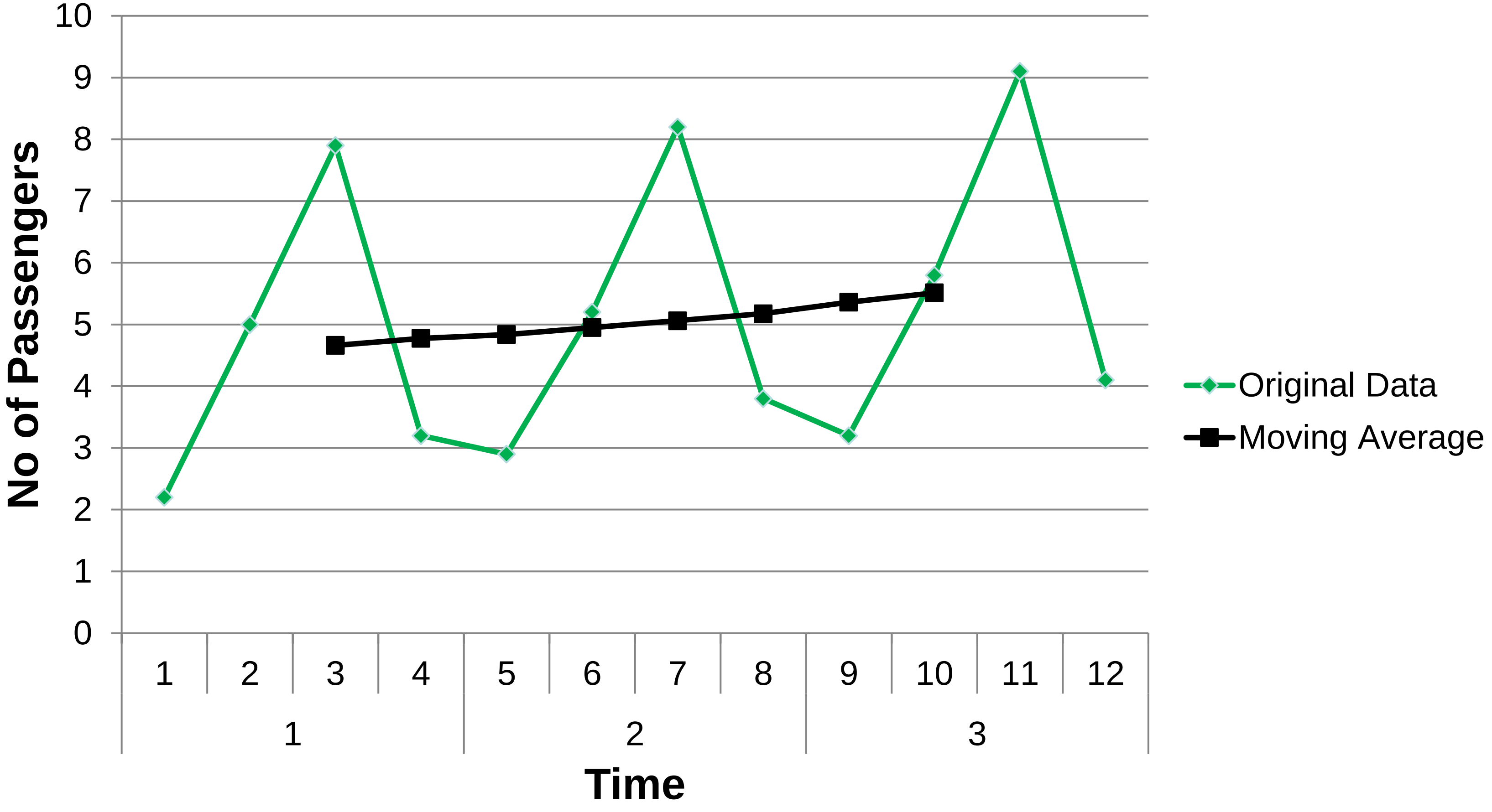
	Year 1				Year 2				Year 3			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
No of passengers (millions)	2.2	5.0	7.9	3.2	2.9	5.2	8.2	3.8	3.2	5.8	9.1	4.1

a) Calculate the seasonal variation using the additive model.

b) Predict the no of passengers for the year 4.

Example (Additive Decomposition Model)

- **Plot of data and trend(MA) value**



Year	Quarter	No of Passenger(million) Y	4 MA	Centered Moving Average	Y – CMA
1	1	2.2			
	2	5	4.575		
	3	7.9	4.75	4.663	
	4	3.2	4.8	4.775	
2	1	2.9	4.875	4.838	
	2	5.2	5.025	4.950	
	3	8.2	5.1	5.063	
	4	3.8	5.25	5.175	
3	1	3.2	5.475	5.363	
	2	5.8	5.55	5.513	
	3	9.1			
	4	4.1			
4	1			Forecast	

Example (Additive Decomposition Model)

Year	Q1	Q2	Q3	Q4	
1					
2					
3					
Total					
Average					
Adjustment factor					
Adjusted Seasonal Variation					

Example (Additive Decomposition Model)

Year	Quarter	No of Passenger(million) y	4 MA	Centered Moving Average (Trend, t)
4	1			Forecast
	2			Forecast
	3			Forecast
	4			Forecast

Multiplicative Decomposition Model

- It is used when the seasonal and irregular fluctuations changes in a specific manner, as a result of the behavior of the trend.
- The amplitude of the seasonality increases with an increasing (decreasing) trend, therefore the components are not independent from each other.
- Most economic time series have seasonal variation that increases with the level of the series. So *multiplicative model* is suitable to them.
- The original time series is a product of its components, so the seasonally adjusted data are calculated as:

$$SA_t = Y_t / S_t$$

Multiplicative Decomposition Model

- Technique to forecast almost similar to additive model. The sum of the seasonal variation = number of periods in a year (average to 1) so that it make no aggregate contribution to the level of the series and the seasonal factors do not capture longer-term (more than 1 year) cycles in a time series.
- A seasonal factor greater than 1 indicates a period in which Y is greater than the yearly average, while a seasonal factor less than 1 indicates a period in which Y is less than the yearly average.

Multiplicative Decomposition Model

- Technique for forecasting

$$Y_{est} = T_{est} \text{ x adjusted seasonal variation}$$

  - where:

Y_{est} = estimated data value

T_{est} = projected trend value

Example (Multiplicative Decomposition Model)

Example 1:

The sales recorded by a supermarket (in thousands $) for the four seasons in the years 2008 to 2010 are as follows: (Correct to three decimal places)

Year	Winter	Spring	Summer	Autumn
2008	100	150	160	140
2009	120	145	170	150
2010	130	180	255	234

a) Using appropriate model to obtain the seasonal variation for the data.

b) Predict the sales for the first two seasons of 2011.

Year	Season	Sales, y	4 MA	Centered Moving Average (Trend,T)	Y/T
2008	W	100			
	Sp	150	137.5		
	Su	160	142.5	140.000	
	A	140	141.25	141.875	
2009	W	120	143.75	142.500	
	Sp	145	146.25	145.000	
	Su	170	148.75	147.500	
	A	150	157.5	153.125	
2010	W	130	178.75	168.125	
	Sp	180	199.75	189.250	
	Su	255			
	A	234			
2011	W			**Forecast**	
	Sp			**Forecast**	

Example (Multiplicative Decomposition Model)

Year	Winter	Spring	Summer	Autumn	
2008					
2009					
2010					
Total					
Average					
Adjustment Factor					
Adjusted Seasonal Variation					

Example (Multiplicative Decomposition Model)

Year	Season	Sales, y	4 MA	Centered Moving Average (Trend,t)
2011	W			**Forecast**
	Sp			**Forecast**

Regression in Forecasting Seasonality and Trend

- Many time series have distinct seasonal pattern. (*For example: room sales are usually highest around summer periods.)*
- Multiple regression models can be used to forecast a time series with seasonal components.
- The use of dummy variables for seasonality is common.
  - Dummy variables needed = total number of seasonality −1
  - For example:

    Quarterly Seasonal: 3 Dummies needed.

    Monthly Seasonal : 11 Dummies needed.
  - The load of each seasonal variable (dummy) is compared to the one which is hidden in intercept.

Regression Model

- Season 1: $S_1 = 1, S_2 = 0, S_3 = 0$
- Season 2: $S_1 = 0, S_2 = 1, S_3 = 0$
- Season 3: $S_1 = 0, S_2 = 0, S_3 = 1$
- Season 4: $S_1 = 0, S_2 = 0, S_3 = 0$

$$\boxed{y_t = \underbrace{\beta_0 + \beta_1 t}_{T_t} + \underbrace{\beta_2 S_1 + \beta_3 S_2 + \beta_4 S_3}_{S_t} + \underbrace{\varepsilon_t}_{\varepsilon_t}}$$

The combination of 0's and 1's for each of the dummy variables at each period indicate the season corresponding to the time series value.

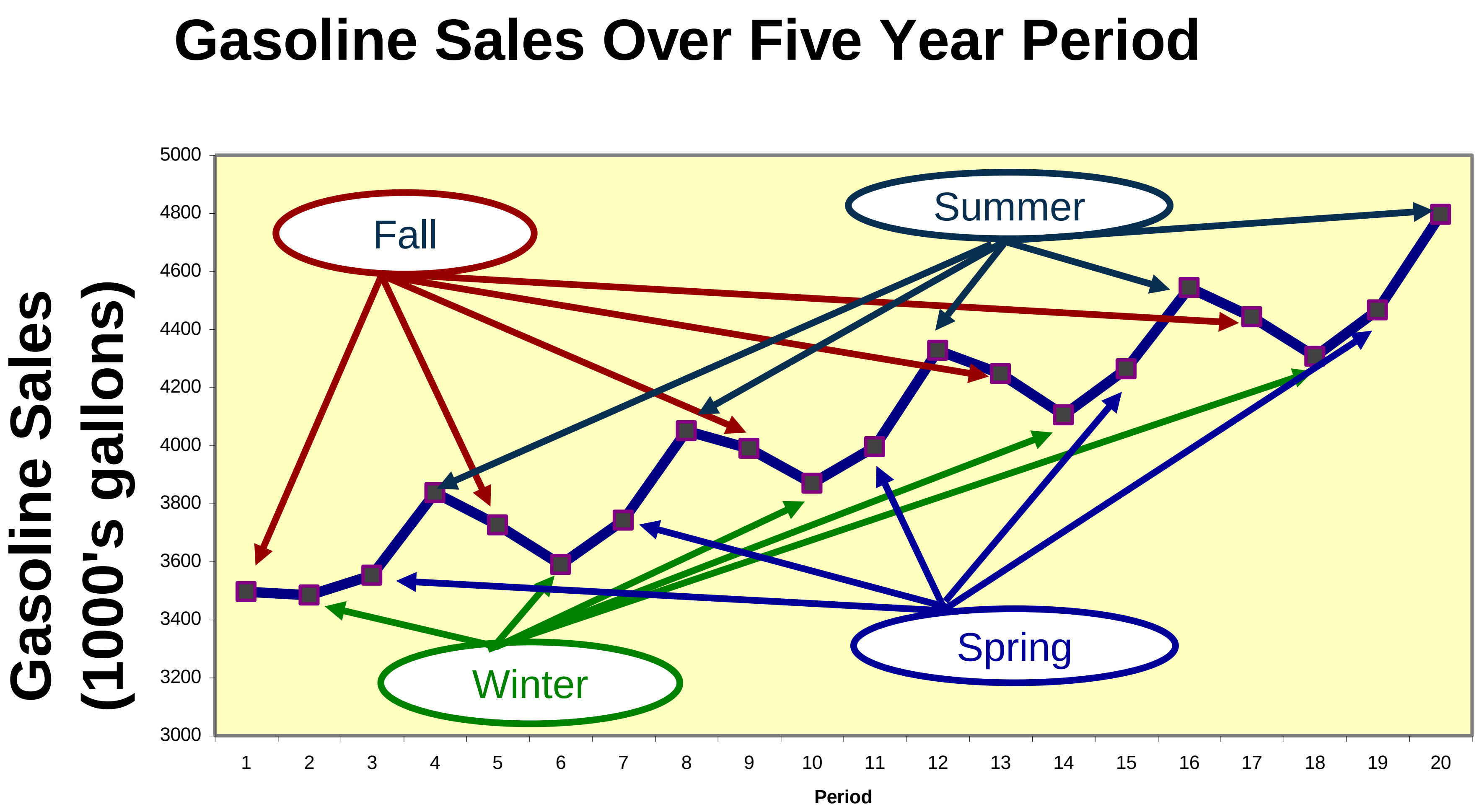
Example (Regression Model)

- Troy owns a gas station in a vacation resort city that has many spring and summer visitors.
  - Due to a steady increase in population, Troy feels that average sales experience long term trend.
  - Troy also knows that sales vary by season due to the vacationers.
- Based on the last 5 years data below with sales in 1000's of gallons per season, Troy needs to predict total sales for next year (periods 21, 22, 23, and 24).

	Year				
Season	1	2	3	4	5
Fall	3497	3726	3989	4248	4443
Winter	3484	3589	3870	4105	4307
Spring	3552	3742	3996	4263	4466
Summer	3837	4050	4327	4544	4795

Example (Regression Model)

- Scatterplot of Time Series
- <u>General Pattern</u>: *Winter less than Fall*, *Spring* more than *Winter*, *Summer* more than *Spring*, *Fall* less than *Summer*



Example (Regression Model)

- There is also apparent long term trend.
- The form of the model then is:

$$\boxed{y_t = \beta_0 + \beta_1 t + \beta_2 F + \beta_3 W + \beta_4 S + \varepsilon_t}$$

Fall **Winter** **Spring**

Example (Regression Model)

Interpretation

- The forecasting regression model is:

$$F_t = 3610.625 + 58.33t - 155F - 323W - 248.27S$$

- b_1: Each additional time sees an average increase of \$58,330 in sales. (Alternative interpretation: The sales increases, on average, by \$233,320 (4 **x** \$58,330) per season year to year)
- b_2: After accounting for the trend, sales in the Fall average about \$155,000 less than <u>Summer</u> sales.
- b_3:
- b_4:

Example (Regression Model)

- The forecasting regression model is:
  $F_t = 3610.625 + 58.33t - 155.006F - 322.938W - 248.269S$

- Forecasts for year 6 are produced as follows:

  F(Year 6, Fall) = 3610.625+58.331(21) – 155.006(1) – 322.938(0) – 248.269(0)

  F(Year 6, Winter) =

  F(Year 6, Spring) =

  F(Year 6, Summer) =

Practical Exercise

You are required to analyse the data below using Seasonal Dummies with Regression and Decomposition Model:

sales.dat – **quarterly** sales data (in $’000), starting **01-01-2007**.

Winter's Exponential Smoothing

- Winter's exponential smoothing model is the second extension of the basic Exponential smoothing model.
- It is used for data that exhibit both trend and multiplicative seasonal pattern.
- It extends Holt's Method to include an estimate for seasonality.
  - α is the smoothing constant for the *level*
  - β is the *trend* smoothing constant - used to remove random error.
  - γ smoothing constant for *seasonality*
- The forecast is modified by multiplying by a seasonal index.

Winter's Exponential Smoothing

- Method Characteristics:
  - Fits a smoothing equation to data
  - Estimating a smoothing equation which minimizes the errors between actual data points and model estimates.
- When to use method
  - Data has trend and seasonality
- When not to use
  - Data does not exhibit trend or seasonality

Winter’s Exponential Smoothing

- The four equations necessary for Winter’s **additive** method are:
  - The exponentially smoothed series:
$$L_t = \alpha(y_t - S_{t-s}) + (1 - \alpha)(L_{t-1} + T_{t-1})$$
  - The trend estimate:
$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$
  - The seasonality estimate:
$$S_t = \gamma(y_t - L_t) + (1 - \gamma)S_{t-s}$$

Winter's Exponential Smoothing

– Forecast m period into the future:

$$F_{t+m} = (L_t + mT_t) + S_{t+m-s}$$

L_t = level of series.

α = smoothing constant for the data.

y_t = new observation or actual value in period t.

β = smoothing constant for trend estimate.

T_t = trend estimate.

γ = smoothing constant for seasonality estimate.

S_t =seasonal component estimate.

m = Number of periods in the forecast lead period.

s = length of seasonality (number of periods in the season)

F_{t+m} = forecast for m periods into the future.

Winter's Exponential Smoothing

- The four equations necessary for Winter's **multiplicative** method are:
  - The exponentially smoothed series:

$$L_t = \alpha\left(\frac{y_t}{S_{t-s}}\right) + (1-\alpha)(L_{t-1} + T_{t-1})$$

  - The trend estimate:

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

  - The seasonality estimate:

$$S_t = \gamma\left(\frac{y_t}{L_t}\right) + (1-\gamma)S_{t-s}$$

Winter's Exponential Smoothing

– Forecast m period into the future:

$$F_{t+m} = (L_t + mT_t)S_{t+m-s}$$

L_t = level of series.
α = smoothing constant for the data.
y_t = new observation or actual value in period t.
β = smoothing constant for trend estimate.
T_t = trend estimate.
γ = smoothing constant for seasonality estimate.
S_t =seasonal component estimate.
m = Number of periods in the forecast lead period.
s = length of seasonality (number of periods in the season)
F_{t+m} = forecast for m periods into the future.

Winter's Exponential Smoothing

- To determine initial estimates of the seasonal indices we need to use at least one complete season's data (i.e. s periods).Therefore, we initialize trend and level at period s.
- Initialize level as:

$$L_s = \frac{1}{p}(y_1 + y_2 + \cdots + y_s)$$

- Initialize trend as:

$$T_p = \frac{1}{p}\left(\frac{y_{p+1} - y_1}{p} + \frac{y_{p+2} - y_2}{p} + \cdots + \frac{y_{p+p} - y_p}{p}\right)$$

- Initialize seasonal indices as:

$$S_1 = \frac{y_1}{L_s}, S_2 = \frac{y_2}{L_s}, \dots, S_p = \frac{y_p}{L_s}$$

Example (Holt-Winters Model)

Determine winter's exponential smoothing forecasts for using $\alpha = 0.1$, $\beta = 0.3$ and $\gamma = 0.4$.

Year	Quarter	Period	Sales
1	1	1	222
	2	2	339
	3	3	336
	4	4	878
2	1	5	443
	2	6	413
	3	7	398
	4	8	1143
3	1	9	695
	2	10	698
	3	11	737
	4	12	1648

Review Questions

Summary / Recap of Main Points

- understand of forecasting techniques, what the most commonly used methods are and how to integrate them into decision making process.
- select an appropriate model for time-related data; learn what the methods can and can’t do, what their strengths and weaknesses are; analyse the data, with or without software and interpret the result.
- solve the model using computer software and interpret the results.

What To Expect Next Week

In Class

Preparation for Class

Performance Evaluation