AQ061-3-M-ODL-TSF Time Series Analysis and Forecasting

Topic 3 – Performance Evaluation

TOPIC LEARNING OUTCOMES

At the end of this topic, you should be able to:

1. make data partitioning.
2. understand the importance of the measurement of errors associated with a forecasting system and how they are used to monitor the forecasting system.
3. use computer software to solve the problems and interpret the results.

Contents & Structure

- Data Partitioning
- Measuring Predictive Accuracy
- Evaluating Forecast Uncertainty

Recap From Last Lesson

- Questions to ask to trigger last week's key learning points

Measuring Predictive Accuracy

- Measure of accuracy use the prediction error that results from predicting the validation period with the model that was trained on the training period.
- The forecast error (e_t) is the difference between the actual value (Y_t) and the forecast value (F_t) at time period t.

$$e_t = Y_t - F_t$$

- Forecast error should be as low as possible.

Measuring Predictive Accuracy

Mean Absolute Error / Deviation (MAE/MAD)

$$MAE / MAD = \frac{\sum_{t=1}^{n} | e_t |}{n}$$

where n = total number of periods

- This gives the magnitude of the average absolute error.

Measuring Predictive Accuracy

Average/Mean Error (AE)

$$AE = \frac{\sum_{t=1}^{n} e_t}{n}$$

- This measure is similar to MAD except that it retains the sign of the errors.
- Gives an indication of whether the forecasts are on average over or under predicting.

Measuring Predictive Accuracy

Mean Percentage Error (MPE)

$$MPE = \frac{1}{n}\sum_{t=1}^{n}\frac{e_t}{Y_t} \times 100\%$$

- Used when it is necessary to determine whether a forecasting method is biased. (consistently forecasting low or high)

Measuring Predictive Accuracy

Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{e_t}{Y_t}\right| \times 100\%$$

- This measure gives a percentage score of how forecasts deviate (on average) from the actual values.
- It is useful for comparing performance across series of data that have different scales.

Measuring Predictive Accuracy

Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2}$$

- This measure has the same units as the data series.

Measuring Predictive Accuracy

Mean Absolute Scaled Error (MASE)

$$MASE = \frac{1}{N}\sum_{i=1}^{N} q_i = \frac{MAE}{MAE*}$$

where **MAE*** is the mean absolute error of the naïve forecast model

- give each error as a ratio compared to a baseline's average error.
- It never gives underdefined or infinite values and so is a good choice for intermittent-demand series (which arise when periods of zero demand in a forecast).
- MASE > 1 implies the forecast does worse than naïve method; MASE < 1 implies that forecast better than a naïve method.

Example

Month	Actual	Forecast
Jan	*1,325*	
Feb	*1,353*	
Mar	*1,305*	
Apr	*1,275*	
May	*1,210*	
Jun	*1,195*	

Exponential Smoothing
($\alpha = 0.2$)

Month	Actual	Forecast
Jan	*1,325*	
Feb	*1,353*	
Mar	*1,305*	
Apr	*1,275*	
May	*1,210*	
Jun	*1,195*	

Forecasting with trend – Linear Trend Model

Exercise

Find MAE, AE, MAPE and RMSE.

Month	Forecast	Actual	Forecast Error
Apr-03	2114	2098	
May-03	2153	2104	
Jun-03	2118	2129	
Jul-03	2229	2223	
Aug-03	2281	2174	
Sep-03	1855	1931	
Oct-03	2101	2121	
Nov-03	2098	2076	
Dec-03	2149	2140	
Jan-04	1920	1831	
Feb-04	1890	1838	
Mar-04	2197	2132	

MAE	
AE	
MAPE	
RMSE	

Data Partitioning

- When we use the same data both to develop the forecasting model and to assess its performance, we introduce bias and overfitting the data.
- Overfitting means that the model is not only fitting the systematic component of the data, but also the noise.
- An over-fitted model is likely to perform poorly on new data.

Data Partitioning

- In cross-sectional data, we typically deal with two or three data partitions:

  a training set,

  a validation set, and

  sometimes as additional test set.
- Partitioning the data is commonly done randomly. But for time series, the series is trimmed into the earlier period (t =1, 2,…n) is set as training set and the later period (t = n+1, n+2,…,n+v) as the validation set.
- In time series partitioning, there is no test period.

Data Partitioning

- The training and validation periods must be recombined into one long series before attempting to forecast future values of the series.
- Then, the chosen method/model is rerun on the complete data.
- Advantages of recombined:
  - The validation periods usually contains the most valuable information (closest in time to the forecasted period).
  - With more data, some models can be estimated more accurately.

Naïve Forecast

- A naïve forecast is simply the most recent values of the series.
- Is simple to understand and implement.
- In other word, at time *t*, the *k*-step ahead naïve forecast is given by

$$F_{t+k} = Y_t$$

- The seasonal naïve forecast is the value from the most recent identical season. For a series with *M* seasons, the formula is

$$F_{t+k} = Y_{t-M+k}$$

Naïve Forecast

- Naïve forecasts are used for two purposes:
  - As the actual forecasts of the series.
  - As a baseline.
- The predictive performance of naïve forecast is evaluated on the validation period, and can examine the corresponding forecast error distribution and create prediction intervals.

Exercise

Split the *quakes.dat* into training (80%) and testing data (20%). Use

- Exponential smoothing
- Holt's method

evaluate their performances.

Evaluating Forecast Uncertainty

- Forecast errors might not be independent and not normally distributed.
- We examine the forecasts errors more to evaluate forecast uncertainty.
- Plotting a histogram or boxplot of the forecasts errors is also very helpful to learn about the expected distribution of forecast errors.

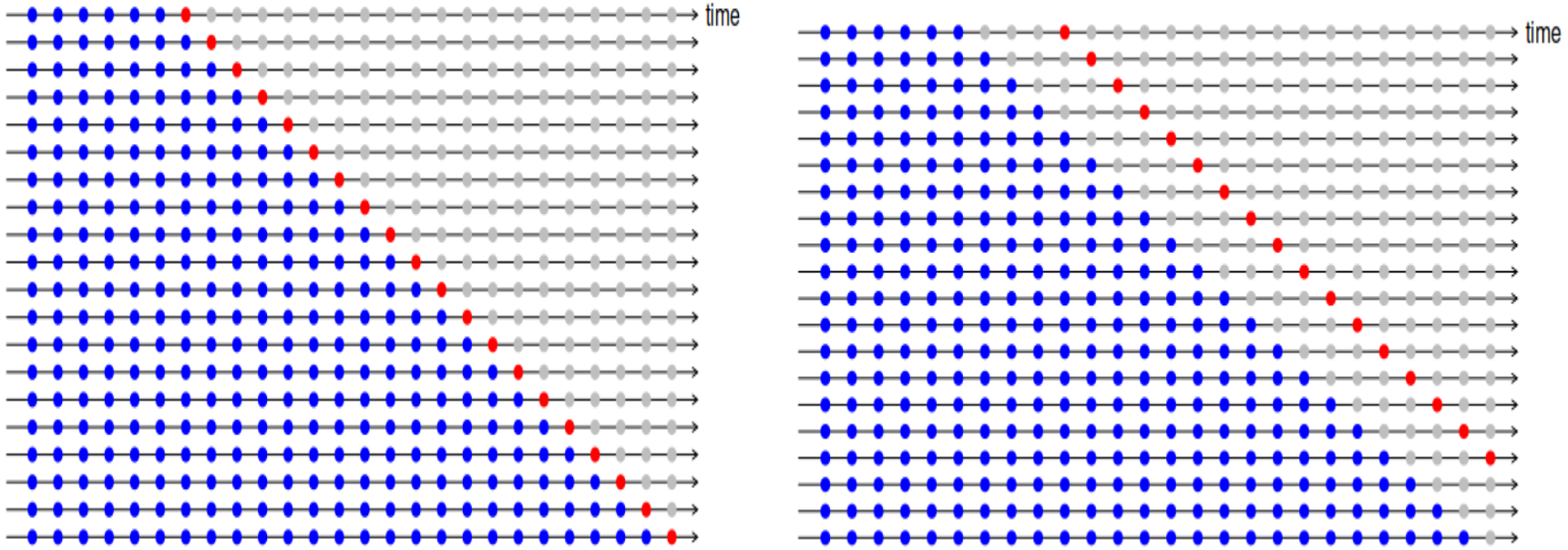
Evaluating Forecast Uncertainty

- Prediction intervals is a forecast range, which has an uncertainty level attached to it.
- A prediction interval is more informative than a single forecast number because it tells about the risk or uncertainty associated with the forecast.
- For example, to forecast the next month sales, the forecast value $50,000 does not tell us how far the forecast can go above or below this number.

Evaluating Forecast Uncertainty

- In contrast, a 95% prediction interval [$40,000, $52,000] tells us that we are 95% certain that the value will fall within this interval. But the point forecast $50,000 is not in the middle of the interval.
- If forecast errors is normally distributed, the prediction intervals can be constructed using $F_t \pm 1.96s$ where s is standard deviation.
- If the forecast errors is not normally distributed, the prediction intervals can be constructed using the 5^{th} and 95^{th} percentiles from the sample forecast errors.

Time Series Cross-Validation



- Start with a small subset of data for training purpose, forecast for the later data points and then checking the accuracy for the forecasted data points. The same forecasted data points are then included as part of the next training dataset and subsequent data points are forecasted.

Review Questions

Summary / Recap of Main Points

What To Expect Next Week

In Class

Preparation for Class

- Box-Jenkins Methodology