AQ061-3-M-ODL-TSF Time Series Analysis and Forecasting

Topic 5 – Volatile Models

TOPIC LEARNING OUTCOMES

At the end of this topic, you should be able to:

1. understand the ARCH and GARCH model estimation.
2. solve the model using computer software and interpret the results.

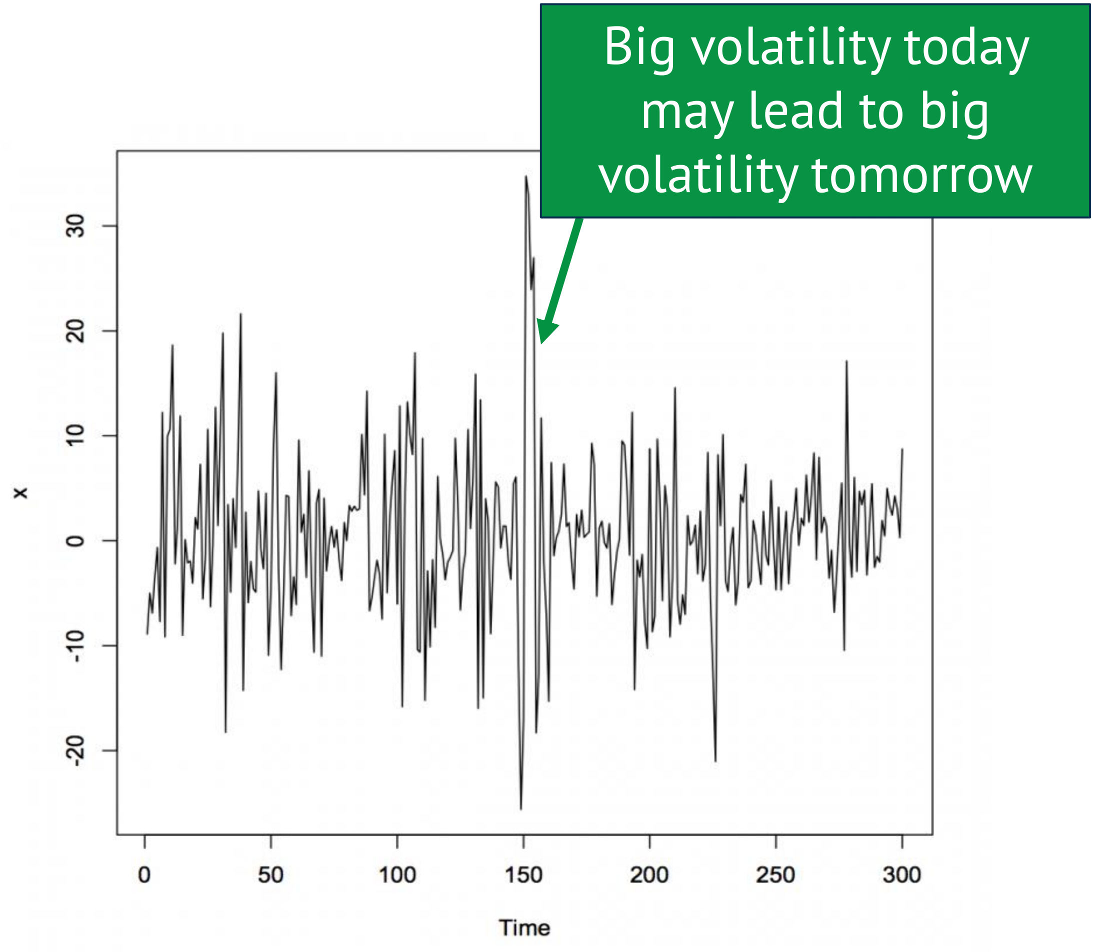
Contents & Structure

- ARCH Model Estimation
- GARCH Model Estimation

Recap From Last Lesson

- Questions to ask to trigger last week’s key learning points

Example of Financial Data



VOLATILITY

- conditional standard deviation of the underlying asset return
- is the degree of **<u>variation of a trading price series over time</u>** as measured by the standard deviation of returns.

The highest the volatility in the market, the highest the uncertainty or return.

Example

Click the link below and choose a historical daily stock price for Samsung. Extract the Adjusted Close data, conduct the following analysis:

https://finance.yahoo.com/quote/005930.KS/history?p=005930.KS

1. Plotting the data
2. Fit an ARIMA model

Problem with Variance

- ARIMA and seasonal ARIMA do not model the change in the variance over time.
- Classically, a time series with modest changes in variance can sometimes be adjusted using a log transform.
- There are some time series where the variance changes consistently over time – increasing and decreasing volatility.
- Changing or unequal variance across the series – heteroskedasticity.
- If the change in variance can be correlated over time, use autoregressive process such as ARCH.

ARCH Model Estimation

- ARCH and GARCH models have become important tools in the analysis of time series data, particularly in financial applications.
- These models are especially useful when the goal of the study is to analyze and forecast the mean and variance of the return, conditional on the past information.
- In finance, there are many reasons why an increase in variance is correlated to a further increase in variance.

ARCH Model Estimation

- It is common that large returns (of any sign) are followed by large returns (of any sign) and vise versa – volatility clustering and due to the fact that information arrives in a short period of time.
- The market becomes volatile whenever big news comes, and it may take several periods for the market to fully digest the news.
- Big volatility (variance) today may lead to big volatility tomorrow.
- Estimation of volatility is one of the most important topics in finance. The volatility of financial assets is a key feature for measuring risk underlying many investment decisions in financial practices.

ARCH Model Estimation

- Engle (1982) suggests the heteroskedastic of conditional variance can be formulated as a linear function of past squared errors. The ARCH(p) model is:

$$y_t = \mu + \varepsilon_t$$
$$\varepsilon_t^2 = z_t^2 \sigma_t$$
$$z_t \sim N(0,1)$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2$$

where $y_t = \log(r_t) - \log(r_{t-1})$ be the log return of an asset, z_t and ε_{t-1} are independent of each other. The residuals can come from autoregression, an ARMA model, or a standard regression model.

ARCH Model Estimation

- The cluster effect, which is the feature of financial data, is captured by σ_t^2 (conditional variance of y_t). The cluster effect indicates that large and small errors tend to cluster together.
- For $p = 1$ and having $\mu_t = 0$, $y_t = z_t\sqrt{\sigma_t^2}$ with $\sigma_t^2 = \omega + \alpha_1 y_{t-1}^2$ becomes the ARCH(1) model.
- Besides, as $y_t = \varepsilon_t = z_t\sqrt{\sigma_t^2}$:

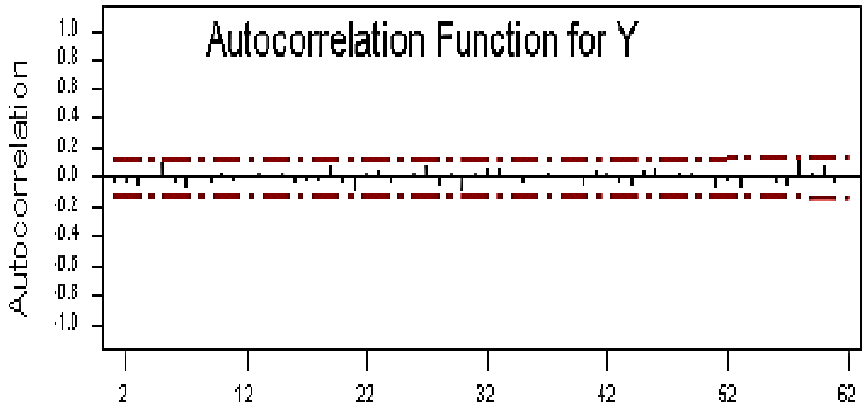
$$y_t^2 = \sigma_t^2 z_t^2 = \omega + \alpha_1 y_{t-1}^2 + \eta_t$$

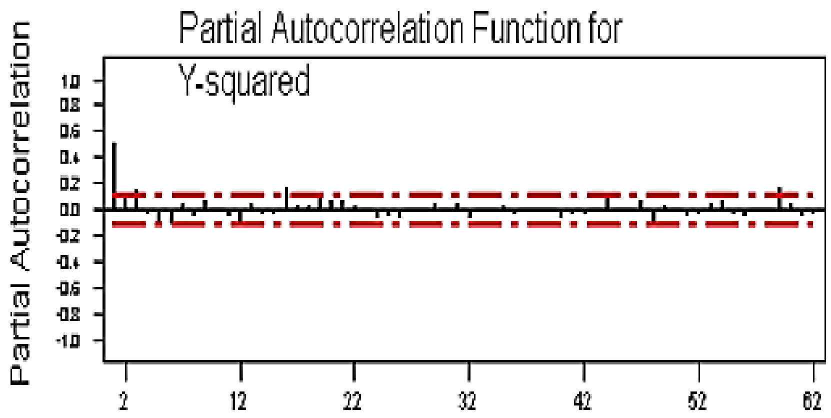
Is a form of AR(1) model in ε_t^2, where $\eta_t = y_t^2 - \sigma_t^2$ is a mean zero uncorrelated with its past.

Identify an ARCH Model

ACF and PACF of both ε_t and ε_t^2

- If y_t appears to be white noise and ε_t^2 appears to be AR(1).
- If the PACF of the ε_t^2 suggests AR(p), then ARCH(p) model.





Test for No-ARCH

- Use LM (Lagrange Multiplier) test for heteroscedasticity.
- Consider the following regression model:

$$\varepsilon_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \varepsilon_{t-2}^2 + \cdots + \text{error}$$

- Under the null hypothesis:

$$H_0\colon \beta_1 = \beta_2 = \cdots = 0$$

If the hypothesis is TRUE, we can say the series have no ARCH effects.

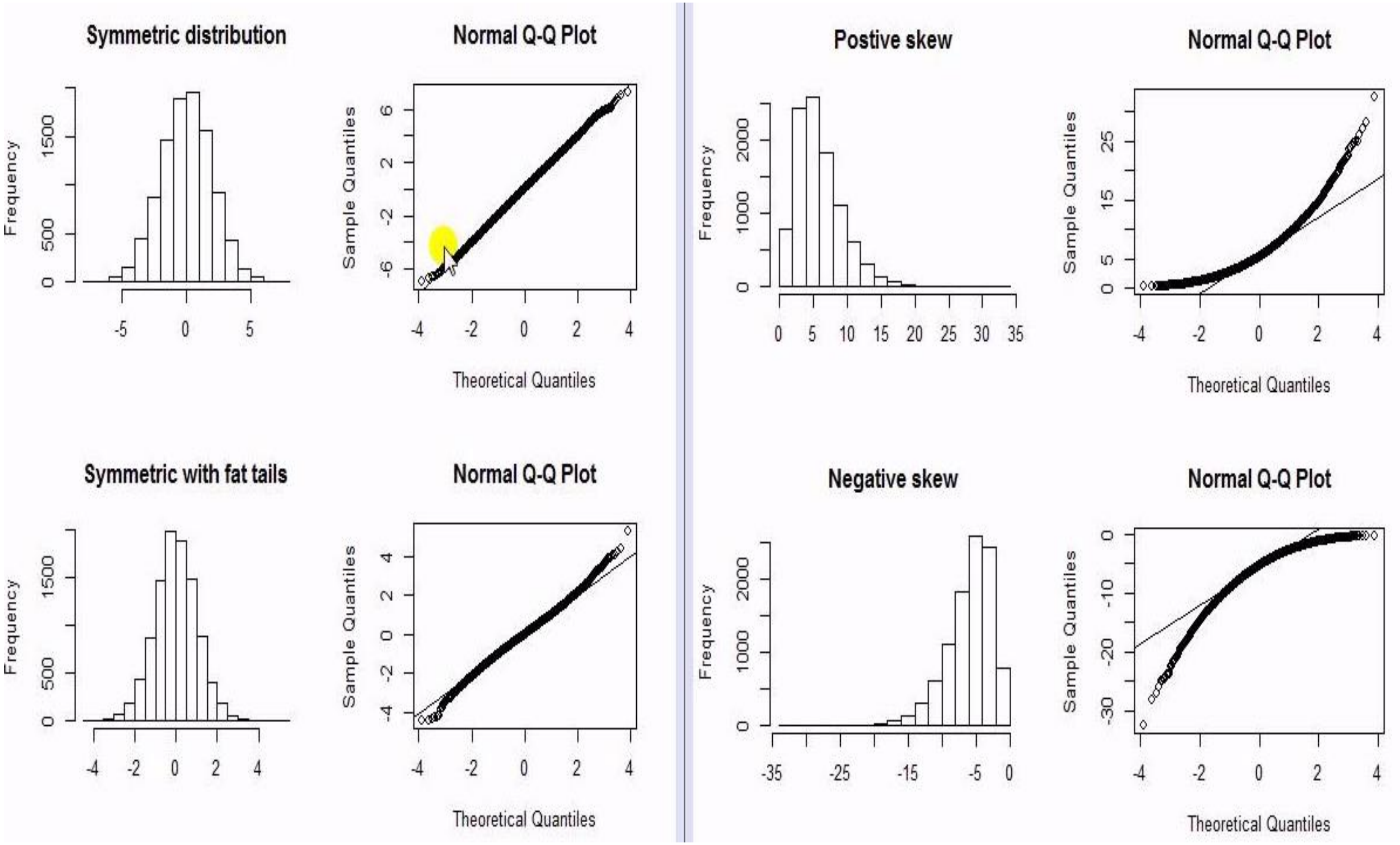
Exercise

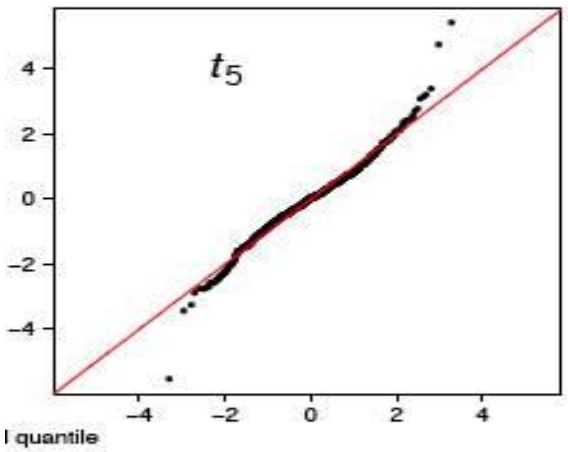
Click the link below and choose a historical daily stock price for a company. Extract the Adjusted Close data, compute the log return and conduct the following analysis:

https://finance.yahoo.com/quote/005930.KS/history?p=005930.KS

1. Plotting the data
2. Fit an ARIMA model and checking for white noise
3. Testing for the ARCH effects
4. Model ARCH model

Type of Patterns in Q-Q Plot





GARCH Model Estimation

- Bollerslev (1986) and Taylor (1986) independently generalised Engle's model to make it more realistic, called as GARCH.
- GARCH is probably the most commonly used financial time series model and has inspired dozens of more sophisticated models.
- GARCH (p, q) model specified the conditional variance to be a function of lagged squared errors and past conditional variance. For q = 0, the GARCH process reduces to an ARCH(p) process. For $p = q = 0$, ε_t is simply white noise.

GARCH Model Estimation

- The GARCH (p, q) process is given as:

$$y_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t = z_t \sqrt{\sigma_t^2}$$

$$z_t \sim N(0,1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-1}^2$$

- Suggested by an ARMA type look to the ACF and PACF of ε_t^2

GARCH Model Estimation

- The sum of the ARCH and GARCH coefficients ($\alpha + \beta$) is very close to one, indicating that volatility shocks are quite persistent.
- Persistent implies that future values of the time series are calculated on the assumption that conditions remain unchanged between current time and future time.

ARMA-GARCH Model

- In the real world, the return process maybe stationary, so we combine the ARMA model and the GARCH model, where we use ARMA to fit the mean and GARCH to fit the variance.
- For example, ARMA(1,1)-GARCH(1,1)

$$y_t = \mu_t + y_{t-1} + \theta\varepsilon_{t-1} + \varepsilon_t$$

where

$$\varepsilon_t = z_t\sqrt{\sigma_t^2}$$

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$$

Exercise

Click the link below and choose a historical daily stock price for Apple company from 28/5/2019 to 22/5/2020. Extract the Adjusted Close data, compute the return and conduct the following analysis:

1. Model ARMA-GARCH model
2. Evaluate the ARMA-GARCH models using AIC and BIC
3. Use the best model to forecast conditional mean, $y_{t+h|t}$ and conditional variance, $\sigma^2_{t+s|t}$.

Review Questions

Summary / Recap of Main Points

What To Expect Next Week

In Class

Preparation for Class

- Class Test