Multilevel Data Analysis

AQ0AQ801-3-M & Version 1.1

Testing and Model Specification

Contents & Structure

- Tests for fixed parameters
- Deviance tests
- Other tests for parameters in the random part
- Model specification

Recap:

- Discuss the different between random intercept model and random slope model.

Learning Outcomes

- **At the end of this topic, You should be able to:**
- **Perform hypothesis testing for HLM**

Key Terms You Must Be Able To Use

- If you have mastered this topic, **you should be able to use the following terms correctly in your assignments and exams:**
- *t-test*
- *F-test*
- *Deviance test*
- *ML and REML estimation*
- *Confidence interval for random part parameters*
- *Model specification*

Leaning Outcome 1

- **Perform hypothesis testing for HLM**

Overview of the Topic

- The first part of this topic discuss hypothesis tests for parameters in the hierarchical linear model, with separate treatments of tests for the parameters in the fixed part and those in the random model.
- The second part is concerned with model specification issues.
- It is assumed that the students have a basic knowledge of statistical hypothesis testing: null and alternative hypothesis, type I and type II errors, significance level, and statistical power.

6.1 Testing and Model Specification

- The null hypothesis testing for a certain regression parameter is 0
- $H_0 \colon \gamma_h = 0$
- Can be tested by a t-test.
- The other parts are quite similar with MLR, except degree of freedom because the presence of the two levels.

- If the total number of level-one units is M and the total number of IV is *r*, then df=M-r-1.
- To test the coefficient of a level-two variable when there are N level-two units and q IV at level-two, df=N-q-1.

- To test the coefficient of the cross-level interaction between level-one variable X and level-two variable Z, when the model contains a total of q other level-two variables also interacting with this variable X, we also use df=N-q-1.
- If the number of degrees of freedom is large enough (say, larger than 40), the t distribution can be replaced by a standard normal distribution.

Example 6.1: Testing within and between group regressions

- Continuing the examples on students in schools of Topic 4 and 5, we now wish to test whether between-group and within-group regressions of language test score on IQ are different from one another, when controlling for socio-economic status (SES).
- A model with a random slope for IQ is used.

- Two models are estimated and presented in Table 6.1 in the next slide.
- The first contains the raw (i.e. grand-mean-centered) IQ variable along with the group mean, the second contains the with-in group deviation variable $\widetilde{IQ}$, defined as
- $\widetilde{IQ}_{ii} = IQ_{ij} - \overline{IQ}_{.j}$
- also together with the group mean.

Table 6.1: Estimates for two models with different between- and within-group regressions.

	Model 1		Model 2	
Fixed effects	Coefficient	S.E.	Coefficient	S.E.
γ_{00} = Intercept	41.15	0.23	41.15	0.23
γ_{10} = Coefficient of IQ	2.265	0.065		
γ_{20} = Coefficient of $\widetilde{IQ}$			2.265	0.065
γ_{30} = Coefficient of SES	0.161	0.011	0.161	0.011
γ_{01} = Coefficient of $\overline{IQ}$	0.647	0.264	2.912	0.262
Random part	Parameter	S.E.	Parameter	S.E.
Level-two parameters:				
$\tau_0^2 = \text{var}(U_{0j})$	9.08	1.12	9.08	1.12
$\tau_1^2 = \text{var}(U_{1j})$	0.197	0.074	0.197	0.074
$\tau_{01} = \text{cov}(U_{0j}, U_{1j})$	–0.815	0.214	–0.815	0.214
Level-one variance:				
$\sigma^2 = \text{var}(R_{ij})$	37.42	0.91	37.42	0.91
Deviance	24,661.3		24,661.3	

- The variable with the random slope is in both models the grand-mean-centered variable IQ.
- To test whether within- and between-group regression coefficients are different, the significance of the group mean $\overline{IQ}$ is tested, while controlling for the effect of the original variable, IQ.

- Table 6.1 shows that only the estimate for $\overline{IQ}$ differs between the two models.
- This is in accordance with Section 4.6: if IQ is variable 1 and $\widetilde{IQ}$ is variable 2, so that their regression coefficients are γ_{10} and γ_{20}, respectively, then the within-group regression coefficient is γ_{10} in Model 1 and γ_{20} in Model 2, while the between-group regression coefficient is $\gamma_{10} + \gamma_{01}$ in Model 1 and γ_{01} in Model 2.

- The models are equivalent representations of the data and differ only in the parametrization.
- The deviances (Explained in Section 6.2) are exactly the same.
- The **within-group regression** and **between-group regressions** are the same if $\gamma_{01} = 0$ in Model 1,that is, there is no effect of the group mean given that the model controls for the raw variable (i.e. the variable without group centering)

- The t-test statistic for testing $H_0: \gamma_{01} = 0$ in Model 1 is equal to 0.647/0.264=2.45.
- This is significant (two-sided p<0.02).
- It may be concluded that within-group and between-group regressions are significantly different.
- The results for Model 2 can be used to test whether the within-group or between-group regressions are 0.

Table 6.1: Estimates for two models with different between- and within-group regressions.

	Model 1		Model 2	
Fixed effects	Coefficient	S.E.	Coefficient	S.E.
γ_{00} = Intercept	41.15	0.23	41.15	0.23
γ_{10} = Coefficient of IQ	2.265	0.065		
γ_{20} = Coefficient of $\widetilde{IQ}$			2.265	0.065
γ_{30} = Coefficient of SES	0.161	0.011	0.161	0.011
γ_{01} = Coefficient of $\overline{IQ}$	0.647	0.264	2.912	0.262
Random part	Parameter	S.E.	Parameter	S.E.
Level-two parameters:				
$\tau_0^2 = \text{var}(U_{0j})$	9.08	1.12	9.08	1.12
$\tau_1^2 = \text{var}(U_{1j})$	0.197	0.074	0.197	0.074
$\tau_{01} = \text{cov}(U_{0j}, U_{1j})$	–0.815	0.214	–0.815	0.214
Level-one variance:				
$\sigma^2 = \text{var}(R_{ij})$	37.42	0.91	37.42	0.91
Deviance	24,661.3		24,661.3	

- The t-statistic for testing the within-group regression is 2.265/0.065 = 34.9, the statistic for testing the between-group regression is 2.912/0.262 = 11.1.
- Both are extremely significant.
- In conclusion, there are positive within-group as well as between-group regressions, and these are different from one another.

Confidence intervals for parameters in the random part

- When constructing confidence intervals for variance parameters it is tempting to use the systematic confidence interval, which for a confidence level of 95% is defined as the estimates plus or minus 1.96 times the standard error.
- This confidence interval, however, is based on the assumption that the parameter estimate is nearly normally distributed, which is doubtful for estimated variances; for example, because there are necessarily nonnegative.

- These confidence intervals are applicable here only when the standard error is quite small compared to the estimate, so that they do not come near the value of 0.
- Generally, they are more safely applied to the standard deviation.
- A symmetric confidence interval for the standard deviation then can be transformed back to a non symmetric confidence interval for the variance, if this is desired.

- Sometimes, also the logarithm of the variance can be used.
- The standard errors of these transformations are related through the approximations

$$S.E.(\ln \hat{\tau}^2) \approx \frac{S.E.(\hat{\tau}^2)}{\hat{\tau}^2} \quad S.E.(\hat{\tau}) \approx \frac{S.E.(\hat{\tau}^2)}{2\hat{\tau}} ---6.2$$

- Where *ln* denotes the natural logarithm.
- If this value is less than 0.1 the distribution of $\hat{\tau}$ will usually be close to a normal distribution; if it is between 0.1 and 0.3 then the symmetric confidence interval still will give a reasonable rough approximation; if it is larger than 0.3 (which means that the number of higher-level units is probably rather low, or the higher-level variance is quite small) the normal approximation itself will break down and the symmetric confidence interval should not be used.

- A better way to construct the confidence interval is based on the so-called profile likelihood.
- This confidence interval consists of those values of the parameter for which twice the logarithm of the profile likelihood is not smaller than twice the logarithm of the maximized likelihood in the ML estimate, minus a value c; this c is the critical value in a chi-squared distribution on 1 degree of freedom (Eg, c=3.84 for a confidence level of 95%)

Example 6.5 Confidence Interval for intercept and slope variances

- In the random intercept model of Example 4.3, the parameter estimates for the two variance components are (S.E = 1.10) for the level-two variance and $\hat{\tau}_0^2 = 8.68$
  (S.E. = 0.96) for the level-one variance $\hat{\sigma}^2 = 40.43$
- The 95% confidence intervals based on the profile likelihood, obtained from R package lme4a, are as follows.

Table 4.4: Estimates for random intercept model with different within- and between-group regressions.

Fixed effect	Coefficient	S.E.
γ_{00} = Intercept	41.11	0.23
γ_{10} = Coefficient of IQ	2.454	0.055
γ_{01} = Coefficient of $\overline{IQ}$ (group mean)	1.312	0.262
Random part	Variance component	S.E.
Level-two variance:		
$\tau_0^2 = \text{var}(U_{0j})$	8.68	1.10
Level-one variance:		
$\sigma^2 = \text{var}(R_{ij})$	40.43	0.96
Deviance	24,888.0	

- The nature of the likelihood-based method implies that the endpoints of the confidence interval for the variances are the squares of those for the standard deviations.
- For the variance, the interval is

$$38.55 \le \sigma^2 \le 42.31$$

$$6.524 \le \tau_0^2 \le 10.836$$

- And for the standard deviation,
- $2.55 \leq \tau_0 \leq 3.29$, $6.21 \leq \sigma \leq 6.51$;

- The symmetric 95% confidence intervals based on the standard errors are defined as the parameter estimates plus or minus 1.96 times the standard errors.

Summary of Main Teaching Points

- Degree of freedom under HLM is different from the normal hypothesis testing due to the nesting structure.
- One may test whether within-group and between-group regressions are significantly different.
- A symmetric confidence interval for the standard deviation then can be transformed back to a non symmetric confidence interval for the variance.

Question and Answer Session

Q & A

What we will cover next

- The End