Operational Research and Optimisation

AQ052-3-M-ORO and VD1

Transportation Model and Its Variants

Topic & Structure of the lesson

- The transportation model
- The assignment model
- The transshipment model
- Software package to be used

Learning Outcomes

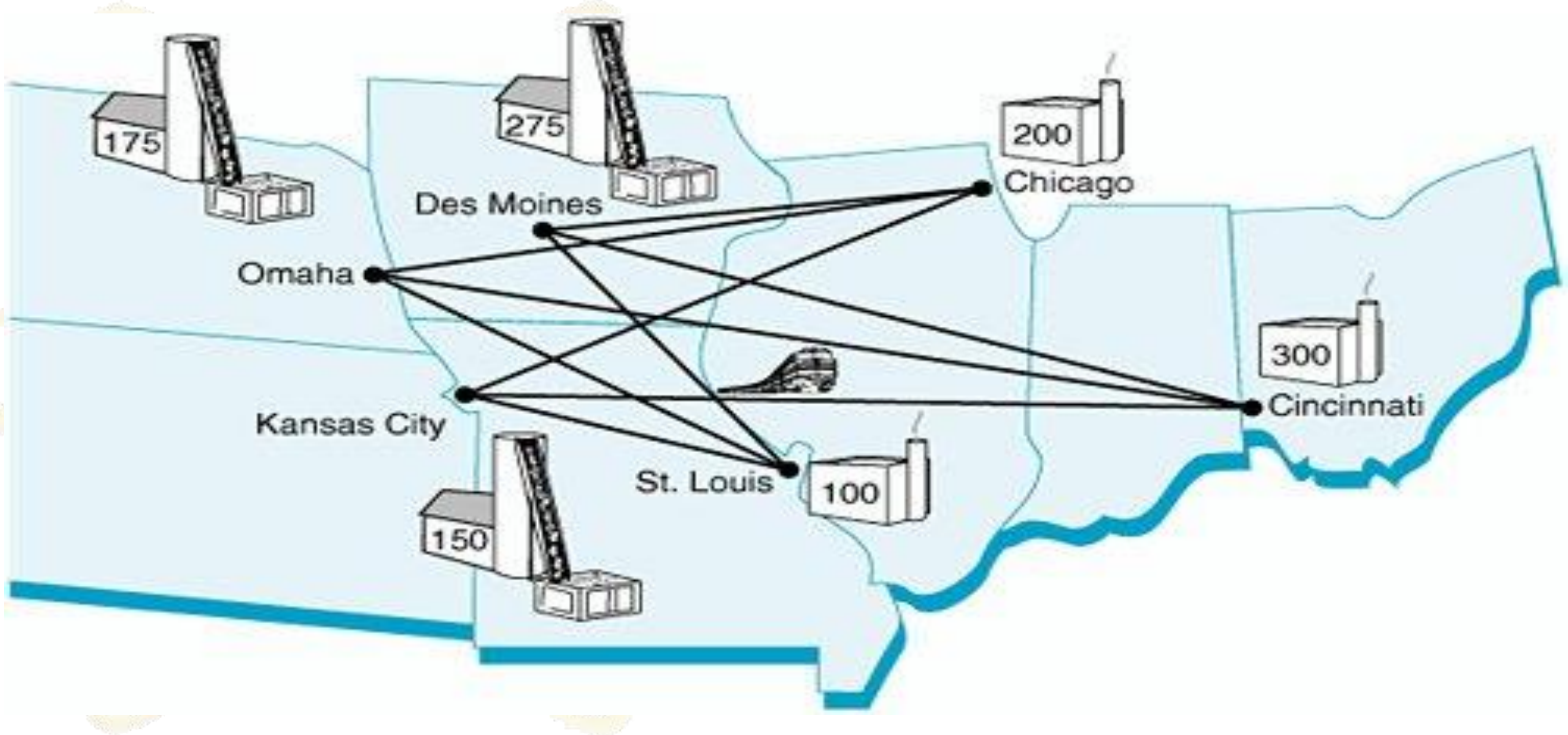
- A the end of this topic, You should be able to model a transportation, assignment and transshipment problems in appropriate forms of linear programming model.

Key Terms you must be able to use

If you have mastered this topic, you should be able to use the following terms correctly in your assignments and exams:

Application:

	Destination	CHICAGO	ST. L OUIS	CINCINNATI
Origin	Kansas City	$6	$8	$10
	Omaha	$7	$11	$11
	Des Moines	$4	$5	$12



Transportation, Assignment, and Transshipment Problems

- A <u>network model</u> is one which can be represented by a set of nodes, a set of arcs, and functions (e.g. costs, supplies, demands, etc.) associated with the arcs and/or nodes.

Transportation, Assignment, and Transshipment Problems

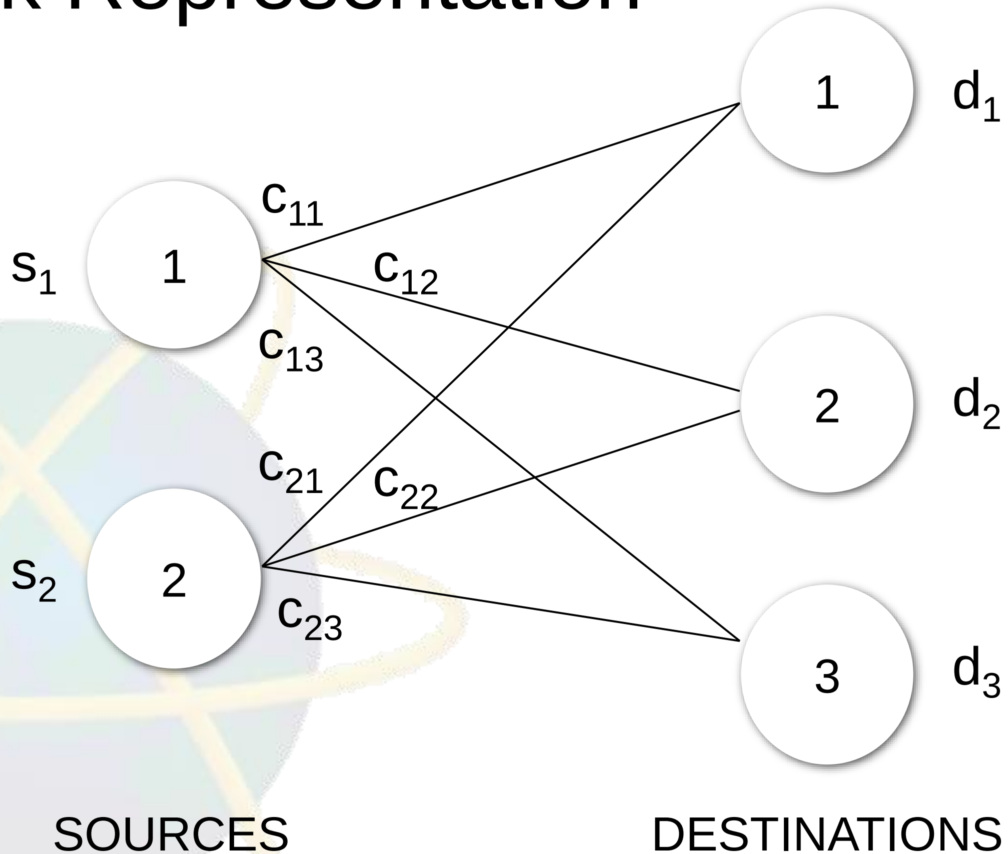
- Each of the three models of this chapter (transportation, assignment, and transshipment models) can be formulated as linear programs and solved by general purpose linear programming Algorithms (simplex method).
- For each of the three models, if the right-hand side of the linear programming formulations are all integers, the optimal solution will be in terms of integer values for the decision variables.

Transportation Problem

- The transportation problem seeks to minimize the total shipping costs of transporting goods from *m* origins or sources (each with a supply s_i) to *n* destinations (each with a demand d_j), when the unit shipping cost from source, *i*, to a destination, *j*, is c_{ij}.
- The network representation for a transportation problem with two sources and three destinations is given on the next slide.

Transportation Problem

- Network Representation



Transportation Problem

- **LP Formulation**

  The linear programming formulation in terms of the amounts shipped from the sources to the destinations, x_{ij} , can be written as:

$$\text{Min} \quad \sum_{i}\sum_{j} c_{ij}x_{ij} \qquad \textit{(total transportation cost)}$$

$$\text{s.t.} \quad \sum_{j} x_{ij} \leq s_i \qquad \text{for each source } i \quad \textit{(supply constraints)}$$

$$\sum_{i} x_{ij} = d_j \qquad \text{for each destination } j \textit{ (demand constraints)}$$

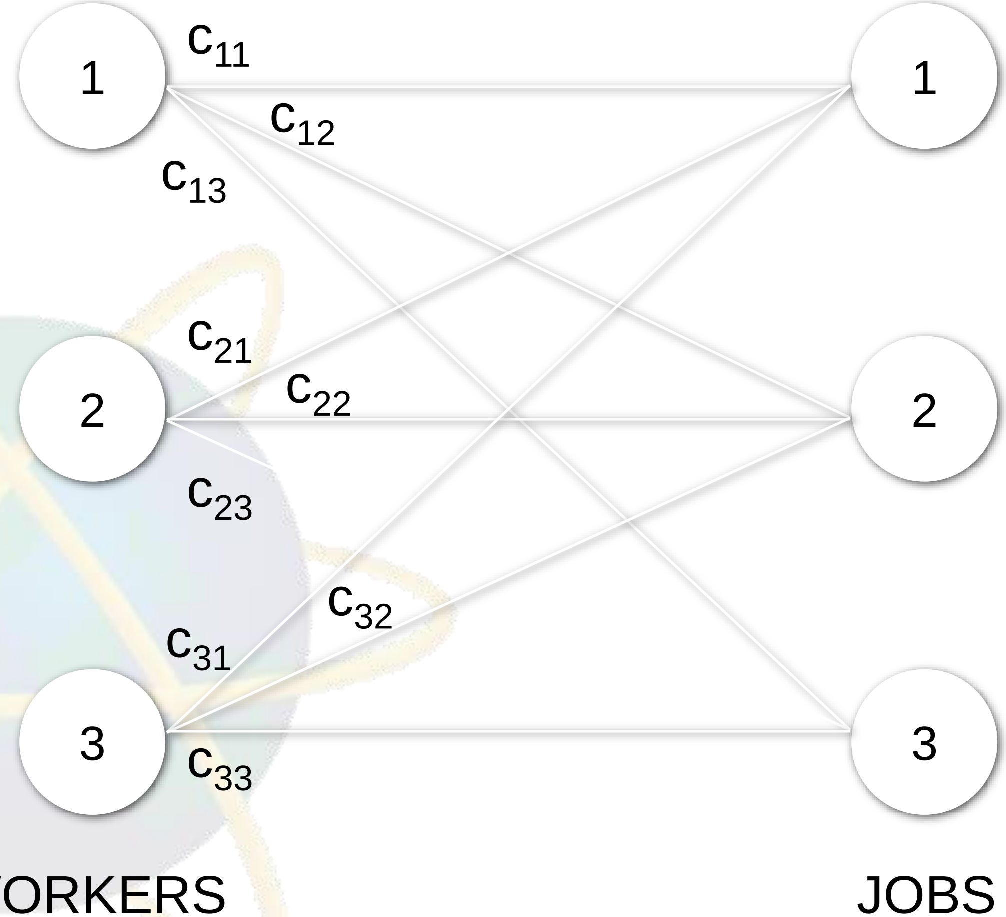
$$x_{ij} \geq 0 \qquad \text{for all } i \text{ and } j \textit{ (nonnegativity constraints)}$$

Assignment Problem

- An <u>assignment problem</u> seeks to minimize the total cost assignment of m workers to m jobs, given that the cost of worker i performing job j is c_{ij}.
- It assumes all workers are assigned and each job is performed.
- An assignment problem is a special case of a <u>transportation problem</u> in which all supplies and all demands are equal to 1; hence assignment problems may be solved as linear programs.
- The <u>network representation</u> of an assignment problem with three workers and three jobs is shown on the next slide.

Assignment Problem

- Network Representation



Assignment Problem

- Linear Programming Formulation

$$\text{Min} \quad \sum_{i}\sum_{j} c_{ij}x_{ij}$$

$$\text{s.t.} \quad \sum_{j} x_{ij} \leq 1 \qquad \text{for each worker } i$$

$$\sum_{i} x_{ij} = 1 \qquad \text{for each job } j$$

$$x_{ij} = 0 \text{ or } 1 \qquad \text{for all } i \text{ and } j.$$

Note: A modification to the right-hand side of the first constraint set can be made if a worker is permitted to work more than 1 job.

Example: Assignment Model

A contractor pays his subcontractors a fixed fee plus mileage for work performed. On a given day the contractor is faced with three electrical jobs associated with various projects. Given below are the distances between the subcontractors and the projects.

		Project		
		A	B	C
Subcontractors	W1	50	36	16
	W2	28	30	18
	W3	35	32	20
	W4	25	25	14

How should the contractors be assigned to minimize total distance?

Example: Assignment Model

- LP Formulation
  - Objective Function

    Minimize total distance:

    $$\text{Min } 50x_{11} + 36x_{12} + 16x_{13} + 28x_{21} + 30x_{22} + 18x_{23} + 35x_{31} + 32x_{32} + 20x_{33} + 25x_{41} + 25x_{42} + 14x_{43}$$

Example: Assignment Model

- LP Formulation
  - Constraints

$$
\begin{aligned}
x_{11} + x_{12} + x_{13} &\leq 1 \quad \text{(no more than one} \\
x_{21} + x_{22} + x_{23} &\leq 1 \quad \text{project assigned} \\
x_{31} + x_{32} + x_{33} &\leq 1 \quad \text{to any one} \\
x_{41} + x_{42} + x_{43} &\leq 1 \quad \text{subcontractor)} \\
x_{11} + x_{21} + x_{31} + x_{41} &= 1 \quad \text{(each project must} \\
x_{12} + x_{22} + x_{32} + x_{42} &= 1 \quad \text{be assigned to just} \\
x_{13} + x_{23} + x_{33} + x_{43} &= 1 \quad \text{one subcontractor)} \\
\text{all } x_{ij} &\geq 0 \quad \text{(non-negativity)}
\end{aligned}
$$

Example: Assignment Model

- Optimal Assignment

Subcontractor	Project	Distance
Westside	C	16
Federated	A	28
Universal	B	25
Goliath	(unassigned)	

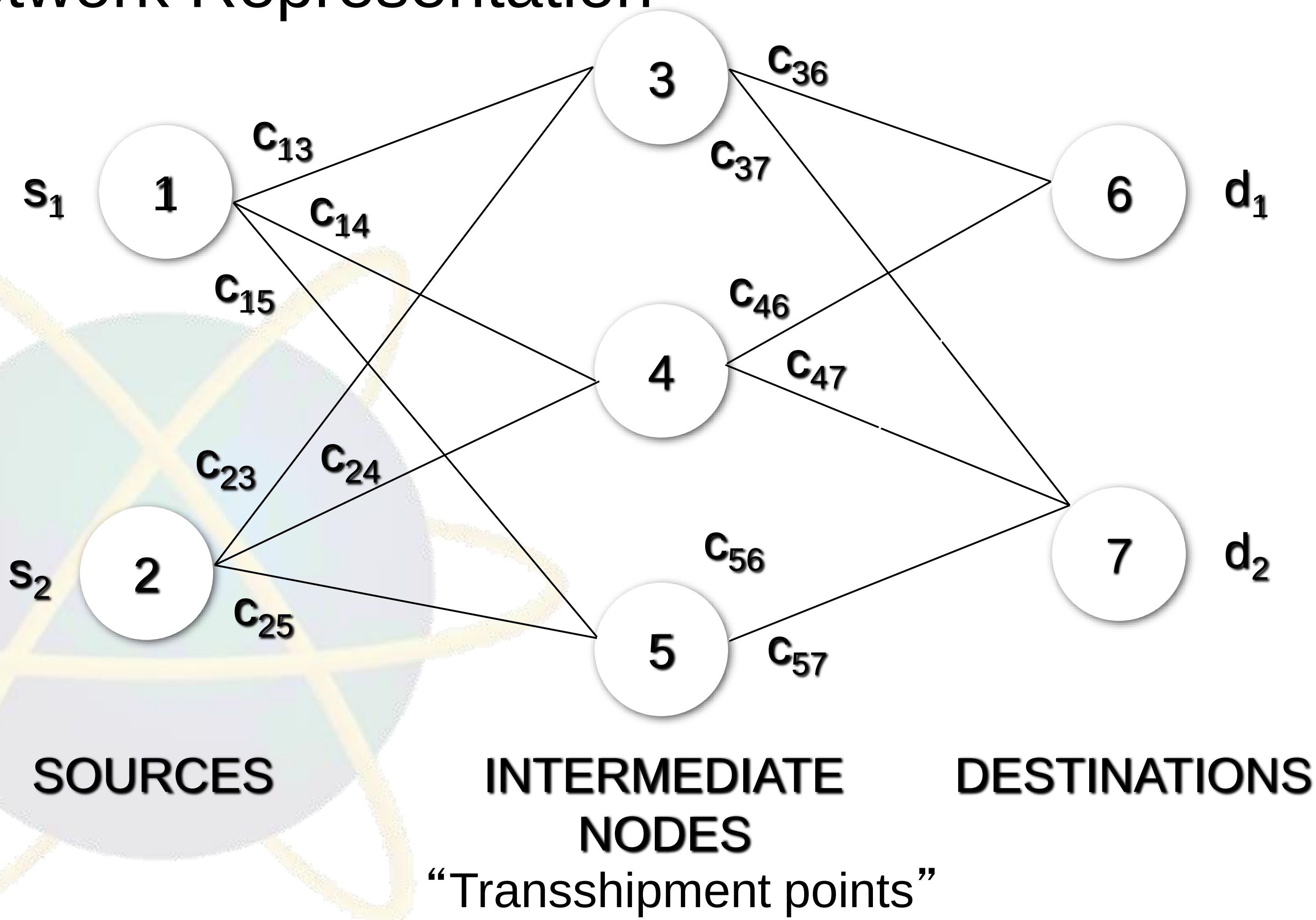
Total Distance = 69 miles

Transshipment Problem

- Transshipment problems are transportation problems in which a shipment may move through intermediate nodes (transshipment nodes)before reaching a particular destination node.
- Transshipment problems can be converted to larger transportation problems and solved by a special transportation program.
- Transshipment problems can also be solved by general purpose linear programming codes.
- The network representation for a transshipment problem with two sources, three intermediate nodes, and two destinations is shown on the next slide.

Transshipment Problem

- Network Representation



Transshipment Problem

- Linear Programming Formulation

  x_{ij} represents the shipment from node *i* to node *j*

$$\text{Min} \quad \sum_i \sum_j c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_j x_{ij} \leq s_i \qquad \text{for each source } i$$

$$\sum_i x_{ik} - \sum_j x_{kj} = 0 \qquad \text{for each intermediate node } k$$

$$\sum_i x_{ij} = d_j \qquad \text{for each destination } j$$

$$x_{ij} \geq 0 \qquad \text{for all } i \text{ and } j$$

Example: Transshipping

Thomas Industries and Washburn Corporation supply three firms (Zrox, Hewes, Rockwright) with customized shelving for its offices. They both order shelving from the same two manufacturers, Arnold Manufacturers and Supershelf, Inc.

Currently weekly demands by the users are 50 for Zrox, 60 for Hewes, and 40 for Rockwright. Both Arnold and Supershelf can supply at most 75 units to its customers.

Example: Transshipping

Because of long standing contracts based on past orders, unit costs from the manufacturers to the suppliers are:

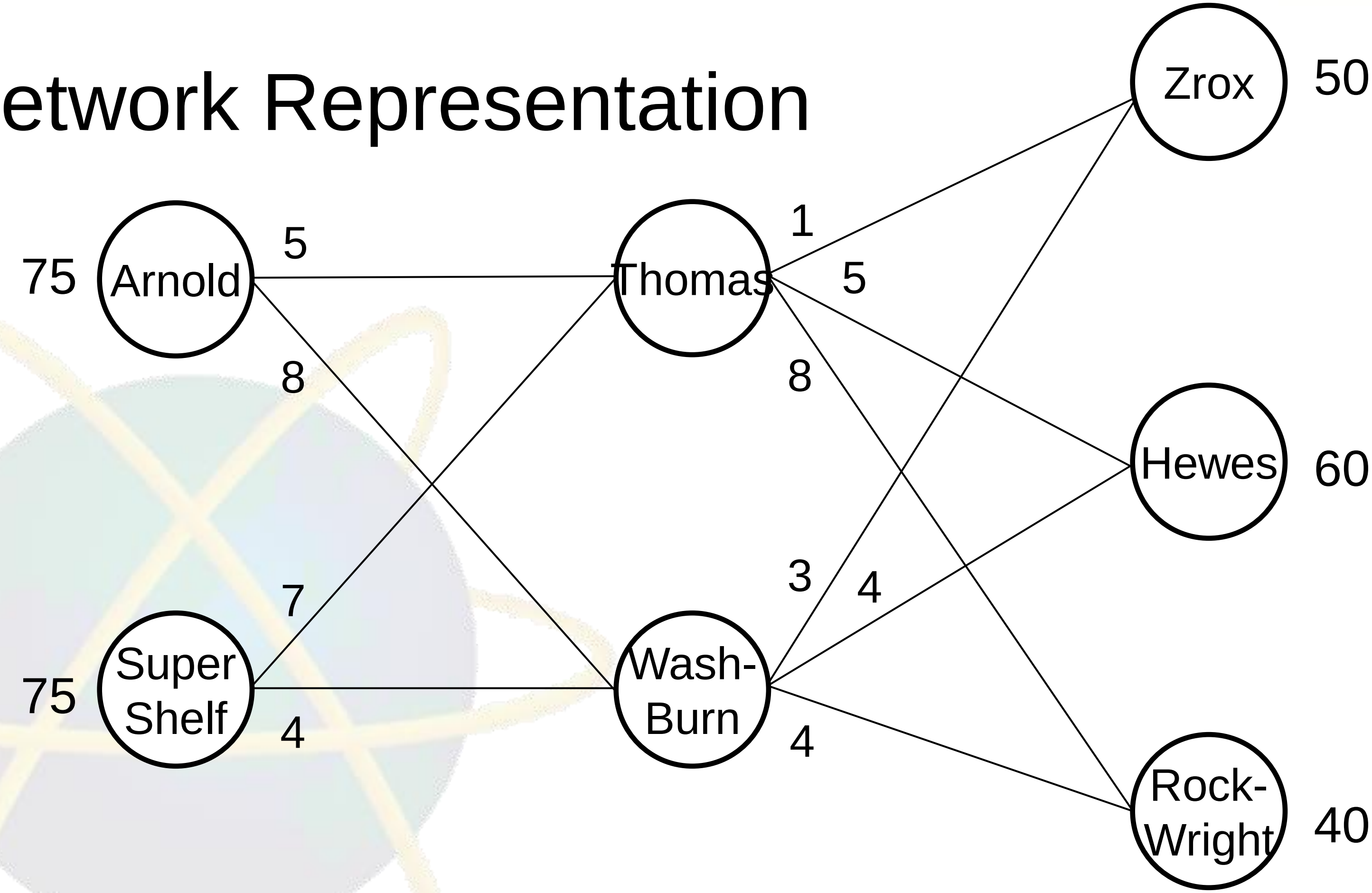
	Thomas	Washburn
Arnold	5	8
Supershelf	7	4

The cost to install the shelving at the various locations are:

	Zrox	Hewes	Rockwright
Thomas	1	5	8
Washburn	3	4	4

Example: Transshipping

- Network Representation



Example: Transshipping

- Linear Programming Formulation
  - Decision Variables Defined

x_{ij} = amount shipped from manufacturer *i* to supplier *j*

x_{jk} = amount shipped from supplier *j* to customer *k*

where i = 1 (Arnold), 2 (Supershelf)

j = 3 (Thomas), 4 (Washburn)

k = 5 (Zrox), 6 (Hewes), 7 (Rockwright)

  - Objective Function Defined

Minimize Overall Shipping Costs:

$$\text{Min} \quad 5x_{13} + 8x_{14} + 7x_{23} + 4x_{24} + 1x_{35} + 5x_{36} + 8x_{37} + 3x_{45} + 4x_{46} + 4x_{47}$$

Example: Transshipping

- Constraints Defined

Amount Out of Arnold: $x_{13} + x_{14} \leq 75$

Amount Out of Supershelf: $x_{23} + x_{24} \leq 75$

Amount Through Thomas: $x_{13} + x_{23} - x_{35} - x_{36} - x_{37} = 0$

Amount Through Washburn: $x_{14} + x_{24} - x_{45} - x_{46} - x_{47} = 0$

Amount Into Zrox: $x_{35} + x_{45} = 50$

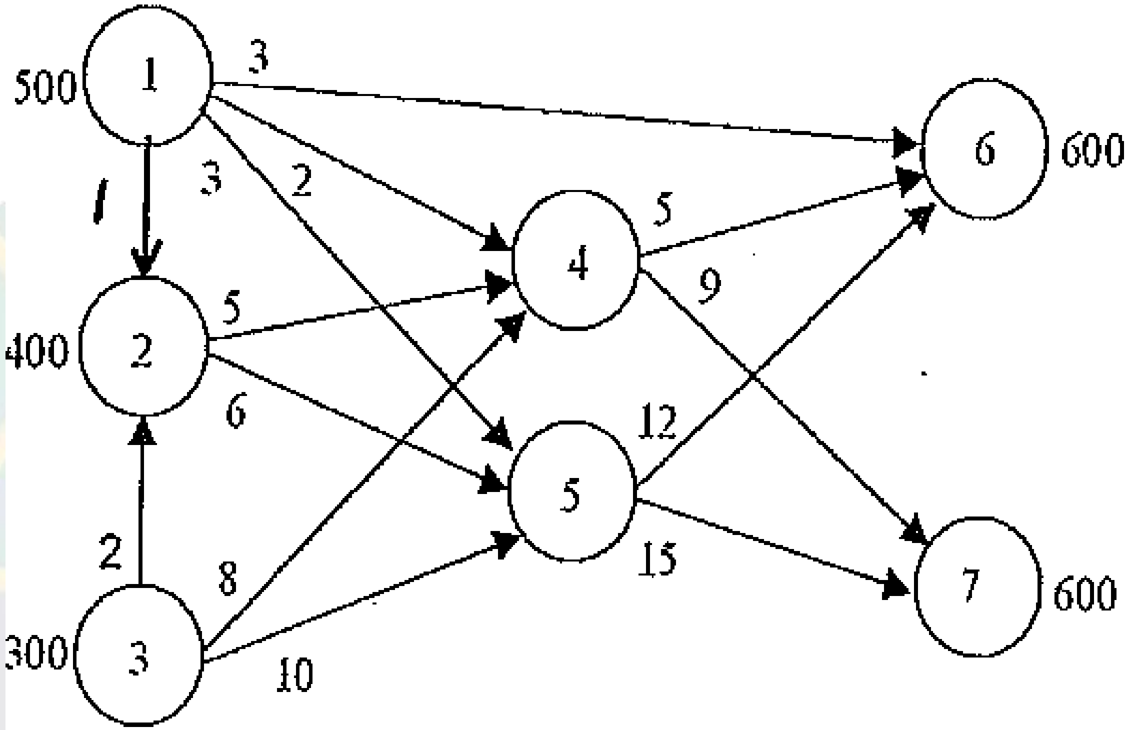
Amount Into Hewes: $x_{36} + x_{46} = 60$

Amount Into Rockwright: $x_{37} + x_{47} = 40$

Non-negativity of Variables: $x_{ij} \geq 0$, for all i and j.

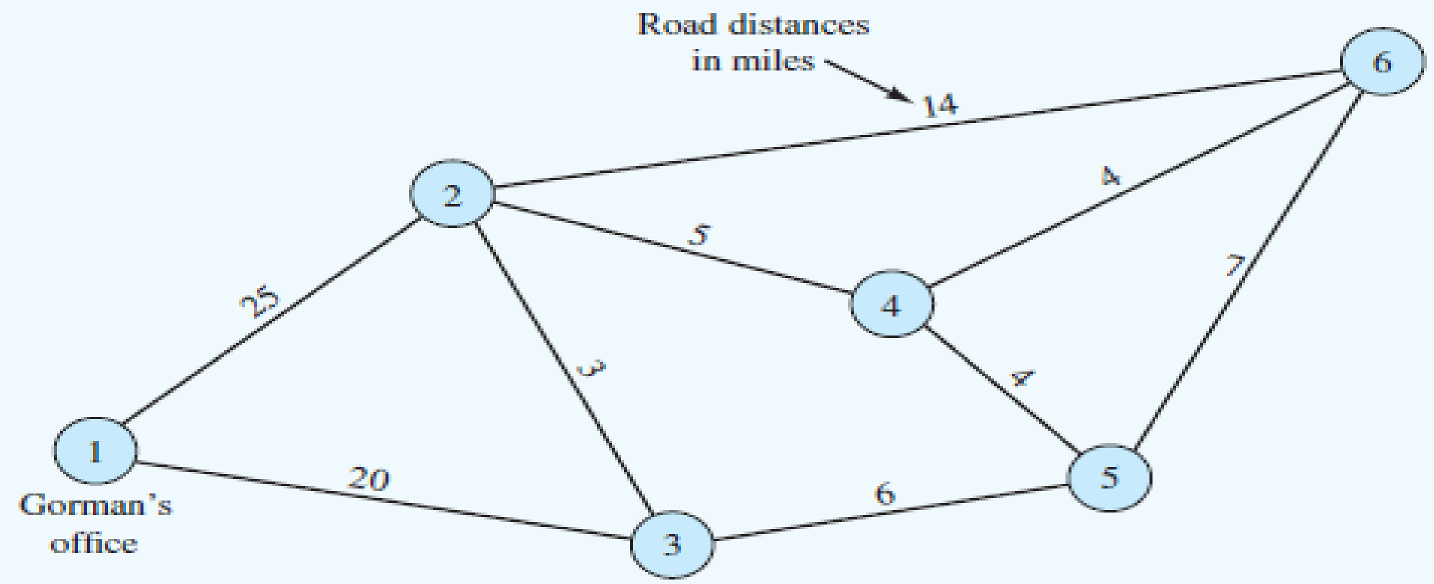
Exercises

Write the linear programming model for the following problem and solve using R or Excel Solver.



Special Case of Transshipment Problem

- Solving a shortest-route problem – from one origin node (node 1) to one destination node (node 6), with four transshipment nodes (node 2, 3, 4 and 5).



Note: (1) The length of each arc is not necessarily proportional to the travel distance it represents.
(2) All roads are two-way; thus, flow may be in either direction.

Quick Review Question

Follow Up Assignment

Summary of Main Teaching Points

Question and Answer Session

Q & A

Next Lesson

Integer Linear Programing