Operational Research and Optimisation

AQ052-3-M-ORO and VD1

Integer Linear Programming

Topic & Structure of the lesson

- Types of Integer Linear Programming
- Graphical and Computer Solution for All-Integer Linear Program
- Application
- Software package

Learning Outcomes

- **A the end of this topic, You should be able to present a real world problem into an integer linear programming model.**

Key Terms you must be able to use

If you have mastered this topic, you should be able to use the following terms correctly in your assignments and exams:

Types of Integer Programming Models

- An LP in which all the variables are restricted to be integers is called an all-integer linear program (ILP).
- The LP that results from dropping the integer requirements is called the LP Relaxation of the ILP.
- If only a subset of the variables are restricted to be integers, the problem is called a mixed-integer linear program (MILP).
- Binary variables are variables whose values are restricted to be 0 or 1. If all variables are restricted to be 0 or 1, the problem is called a 0-1 or binary integer linear program.

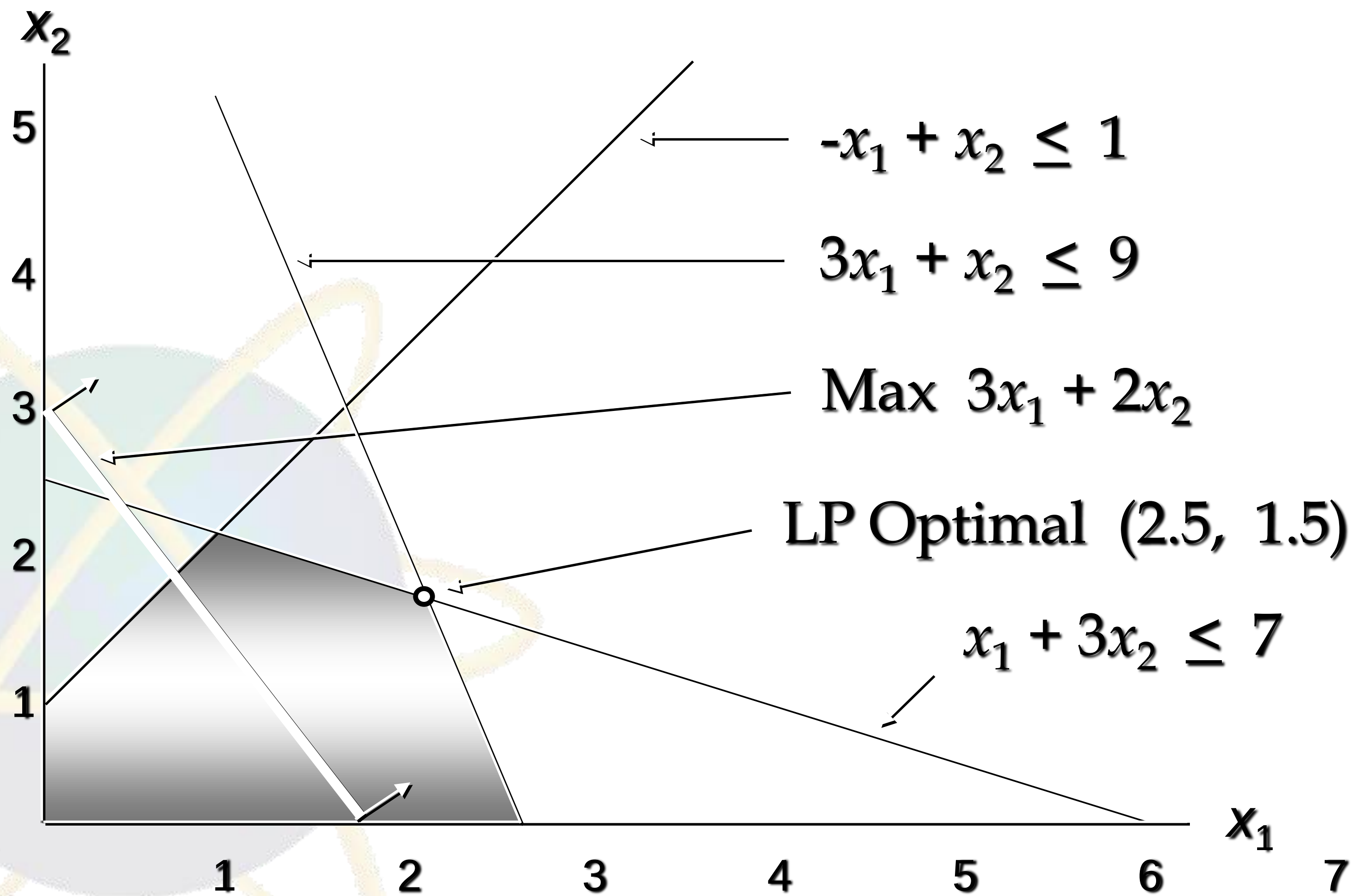
Example: All-Integer LP

- Consider the following all-integer linear program:

$$
\begin{aligned}
\text{Max} \quad & 3x_1 + 2x_2 \\
\text{s.t.} \quad & 3x_1 + x_2 \leq 9 \\
& x_1 + 3x_2 \leq 7 \\
& -x_1 + x_2 \leq 1 \\
& x_1, x_2 \geq 0 \text{ and integer}
\end{aligned}
$$

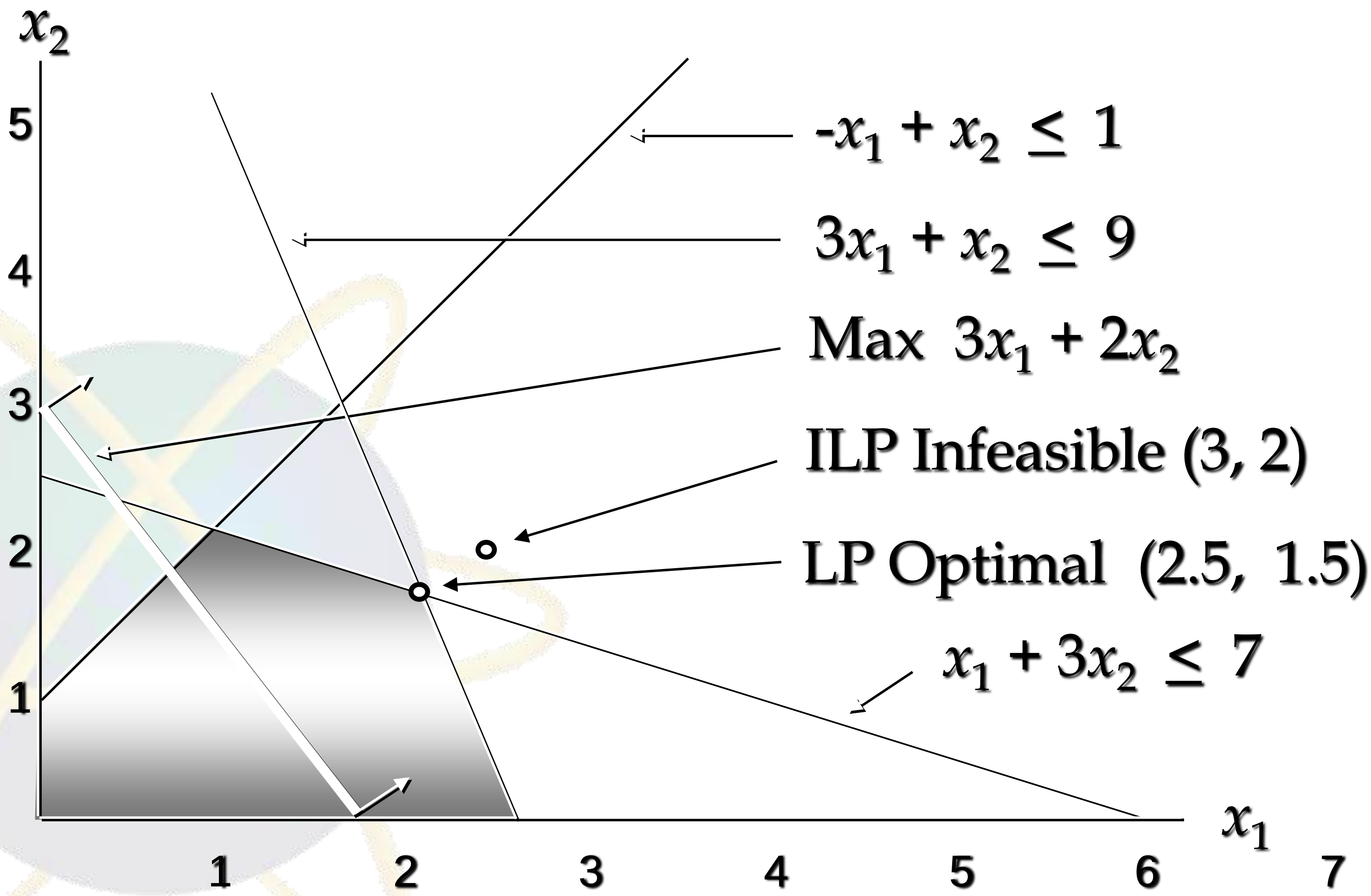
Example: All-Integer LP

- LP Relaxation



Example: All-Integer LP

- Rounded Up Solution



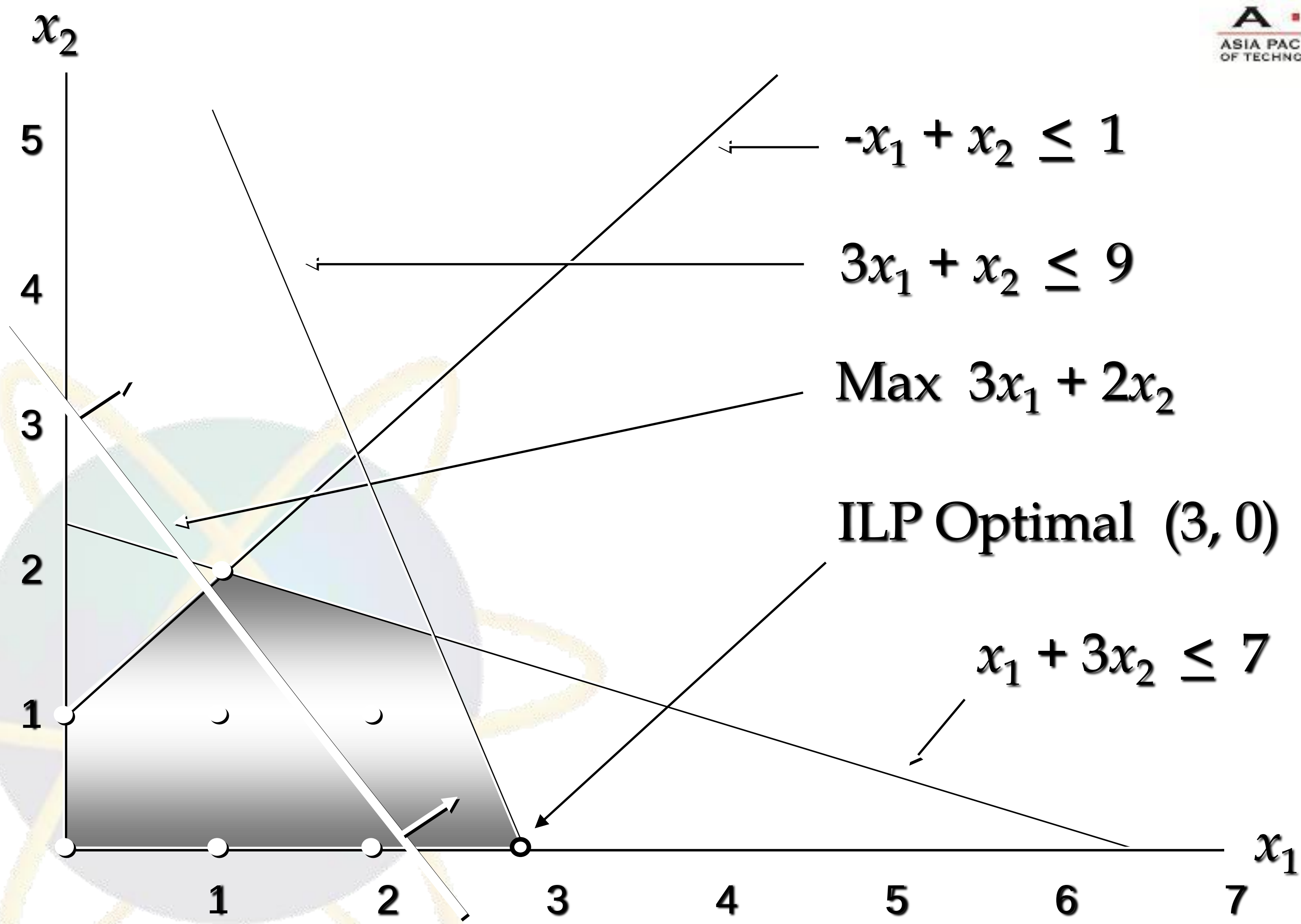
Example: All-Integer LP

- **Complete Enumeration of Feasible ILP Solutions**

  There are eight feasible integer solutions to this problem:

	x_1	x_2	z	
1.	0	0	0	
2.	1	0	3	
3.	2	0	6	
4.	3	0	9	← optimal solution
5.	0	1	2	
6.	1	1	5	
7.	2	1	8	
8.	1	2	7	

Example: All-Integer LP



Application

- Capital Budgeting
- Distribution System Design
- Store locations – set covering problem

Capital Budgeting

The Ice-Cold Refrigerator Company is considering investing in several projects that have varying capital requirements over the next four years. Faced with limited capital each year, management would like to select the most profitable projects. The estimated net present value (net cash flow) for each project, the capital requirements, and the available capital over the four-year period is given.

Table

	Project				
	Plant Expansion	**Warehouse Expansion**	**New Machinery**	**New Product Research**	**Total Capital Available**
Present Value	$90,000	$40,000	$10,000	$37,000	
Year 1	$15,000	$10,000	$10,000	$15,000	$40,000
Year 2	$20,000	$15,000		$10,000	$50,000
Year 3	$20,000	$20,000		$10,000	$40,000
Year 4	$15,000	$5,000	$4,000	$10,000	$35,000

Decision variable

P = 1 if the plant expansion project is accepted

W = 1 if the warehouse expansion project is accepted

M = 1 if the new machinery project is accepted

R = 1 if the new product research project is accepted

$$
\begin{array}{llr}
\text{Max} & 90P + 40W + 10M + 37R & \\
\text{s.t.} & 15P + 10W + 10M + 15R \le 40 & \text{(Year 1 capital)} \\
 & 20P + 15W \qquad\qquad + 10R \le 50 & \text{(Year 2 capital)} \\
 & 20P + 20W \qquad\qquad + 10R \le 40 & \text{(Year 3 capital)} \\
 & 15P + 5W + 4M + 10R \le 35 & \text{(Year 4 capital)} \\
 & P, W, M, R = 0, 1 &
\end{array}
$$

- Optimal solution:

  P = 1, W = 1, M = 1, R = 0

  total estimated net present value of $140,000

- The values of the slack variables show that the company will have $5,000 remaining in year 1, $15,000 remaining in year 2, and $11,000 remaining in year 4.

Questions!!

- Write the constraint if:

(i) Two projects P, W and M must be undertaken.

(ii) Project M and R must be undertaken simultaneously.

(iii) Project M or R must be undertaken, but not both.

(iv) When projects M and R undertaken, project P must be undertaken.

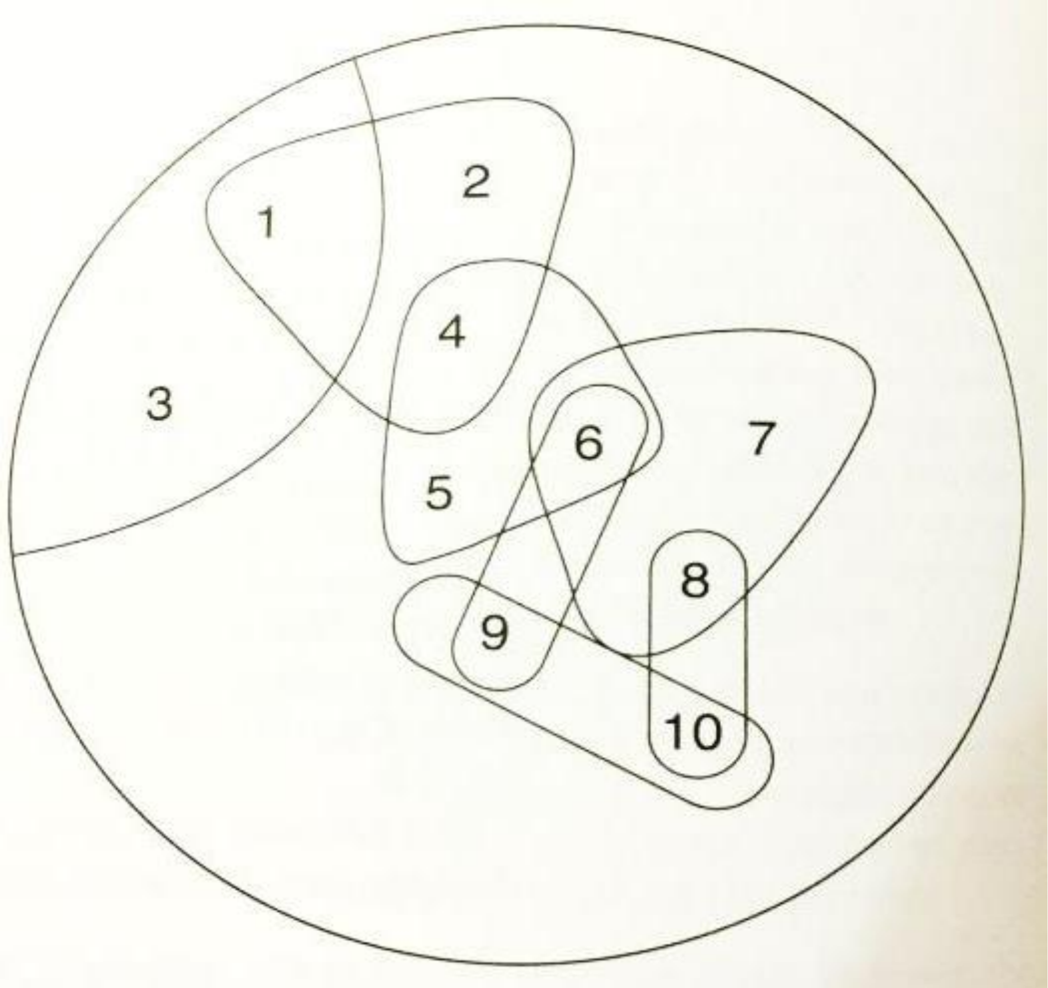
A Set Covering Example

Company ABC has brought out a competing grocery store chain. However, it now has too many stores in close proximity to each other in certain city. In Cheras, the chain has 10 stores and it does not want any stores closer than 2 miles to each other. Following are the monthly revenue (in thousands) from each store and a map showing the general proximity of the stores. Stores within 2 miles of each other are circled.

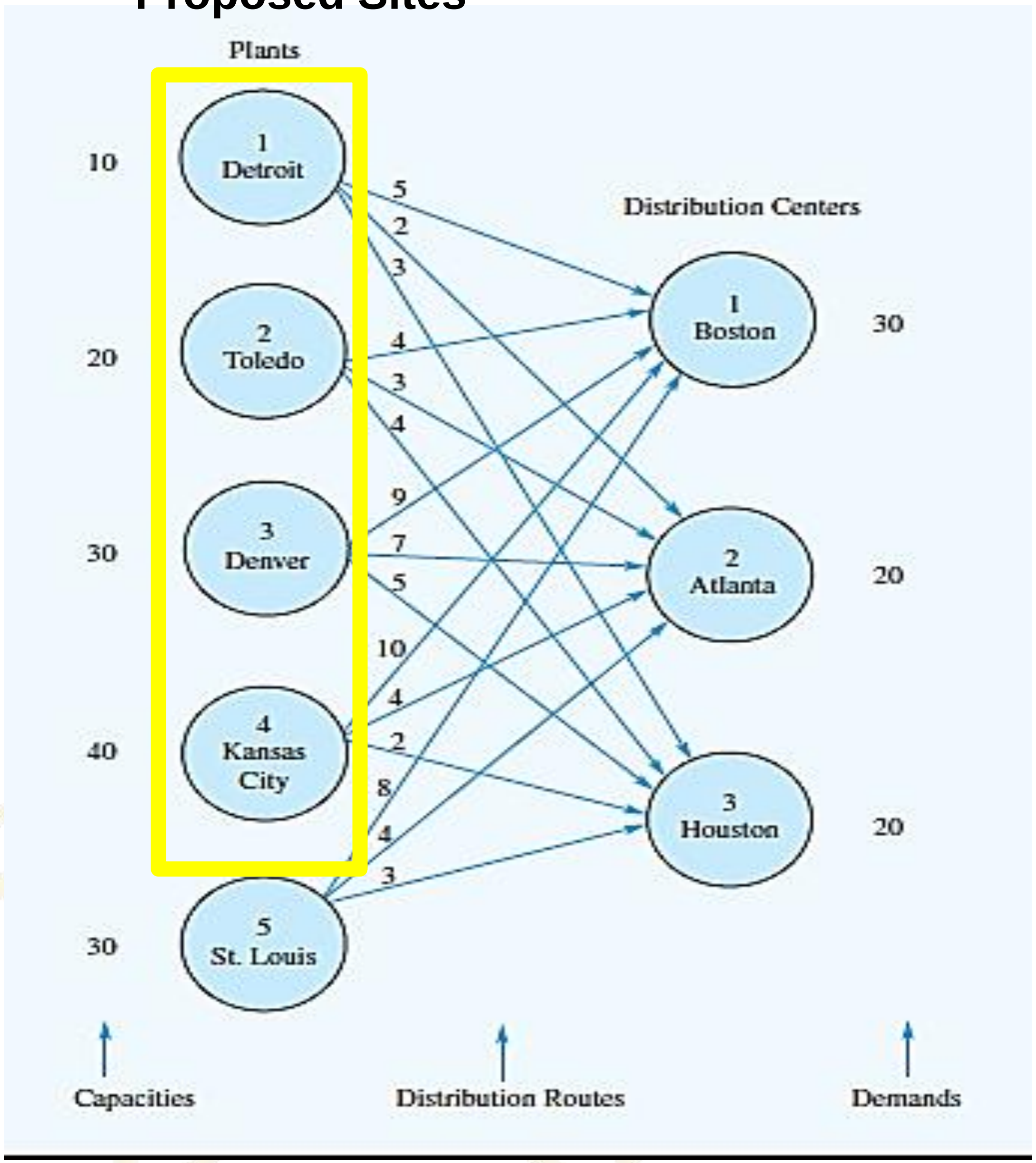
Develop and solve an integer linear programming model to determine which stores should keep open in Cheras.

Continue…

Store	Monthly revenue
1	$127
2	$83
3	$165
4	$96
5	$112
6	$88
7	$135
8	$141
9	$117
10	$94



Proposed Sites



Proposed transportation system

Distribution System Design

Proposed Sites

Plant site	Destination			Capacity (1000s)
	Boston	Atlanta	Houston	
Detroit	5	2	3	**10**
Toledo	4	3	4	**20**
Denver	9	7	5	**30**
Kansas City	10	4	2	**40**
St. Louis	8	4	3	**30**
Demand (1000s)	**30**	**20**	**20**	

- Given the estimated annual fixed cost for the new plants (in thousands) are

Detroit	$175
Toledo	$300
Denver	$375
Kansas City	$500

- Supposedly there is no plant in Detroit, Toledo, Denver and Kansas City, formulate a mixed-integer programming to decide which plants to build.

Quick Review Question

Follow Up Assignment

Summary of Main Teaching Points

Question and Answer Session

Q & A

Next Lesson

Goal Programming