Operational Research and Optimisation

AQ052-3-M-ORO and VD1

Markov Process

ASIA PACIFIC UNIVERSITY
OF TECHNOLOGY & INNOVATION

Topic & Structure of the lesson

- Transition Probabilities
- Steady-State Probabilities
- Absorbing States
- Transition Matrix with Submatrices
- Fundamental Matrix

Learning Outcomes

- A the end of this topic, You should be able to
  - model a problem in a Markov process.
  - solve the problem and interpret the results.

Key Terms you must be able to use

If you have mastered this topic, you should be able to use the following terms correctly in your assignments and exams:

Example

A student has class on Monday, Wednesday and Friday. If he goes to class on one day, he goes to the next class with probability 0.5. If he doesn't go to class that day, he goes to the next class with probability 0.75.

Find the probability he goes to class on Friday, if he didn't go to class the preceding Monday. Find the probability he will show up for the class the following Monday if he goes to class on Monday.

Markov Processes

- Markov process models are useful in studying the evolution of systems over repeated trials or sequential time periods or stages.
- They have been used to describe the probability that:
  - a machine that is functioning in one period will continue to function or break down in the next period.
  - A consumer purchasing brand A in one period will purchase brand B in the next period.

Example

Customers in a certain city are continually switching the brand of body shampoo they buy. If a customer is now using brand A, the probability he will use brand A next week is 0.5, that he switches to brand B is 0.2 and that he switches to brand C is 0.3. If he now uses brand B, the probability he uses B next week is 0.6 and the probability that he switches to C is 0.4. If he now uses brand C, the probability he uses C next week is 0.4, that he switches to A is 0.2 and to B is 0.4. Assume the process is a Markov chain.

a) Find the probability a customer now using brand A will be using brand B in two weeks.

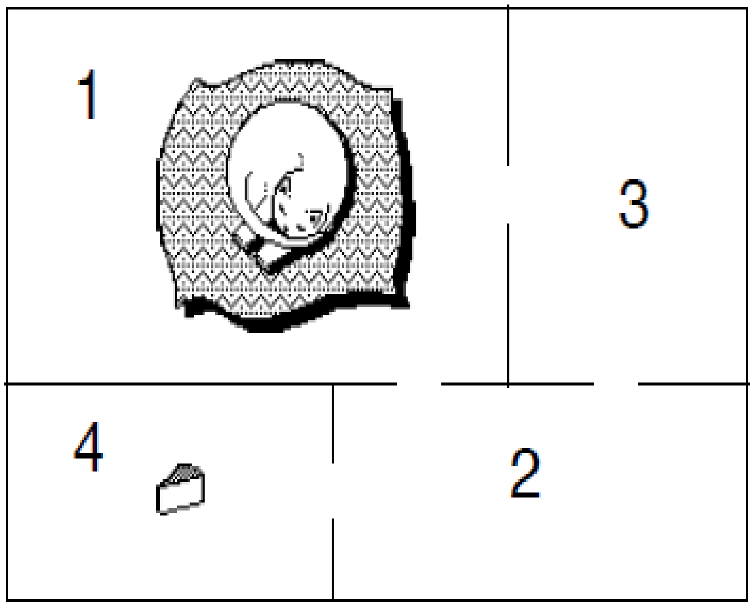
b) Find the steady-state probabilities for each brand A, B and C.

Absorbing States

- An <u>absorbing state</u> is one in which the probability that the process remains in that state once it enters the state is 1.
- If there is more than one absorbing state, then a steady-state condition independent of initial state conditions does not exist.

Example

A rat is placed in the maze of the figure. In room 1 is a cat and in room 4 is a piece of cheese. If the rat enters either of these rooms he does not leave it. If he is in one of the other rooms, each time period he chooses at random one of the doors of the room he is in and moves into another room.



Transition Matrix with Submatrices

- If a Markov chain has both absorbing and nonabsorbing states, the states may be rearranged so that the transition matrix can be written as the following composition of four submatrices: *I*, *0*, *R*, and *Q*:

$$\left[\begin{array}{c|c} I & 0 \\ \hline R & Q \end{array}\right]$$

Transition Matrix with Submatrices

I = an identity matrix indicating one always remains in an absorbing state once it is reached

0 = a zero matrix representing 0 probability of transitioning from the absorbing states to the nonabsorbing states

R = the transition probabilities from the nonabsorbing states to the absorbing states

Q = the transition probabilities between the nonabsorbing states

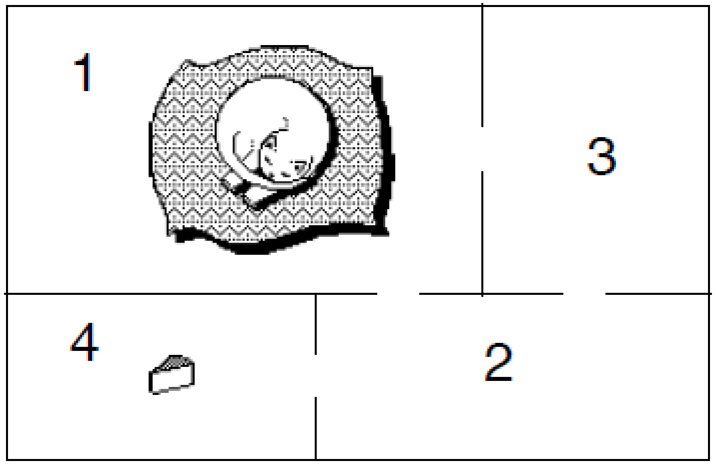
Fundamental Matrix

- The fundamental matrix, *N*, is the inverse of the difference between the identity matrix and the *Q* matrix:

$$N = (I - Q)^{-1}$$

Example

A rat is placed in the maze of the figure. In room 1 is a cat and in room 4 is a piece of cheese. If the rat enters either of these rooms he does not leave it. If he is in one of the other rooms, each time period he chooses at random one of the doors of the room he is in and moves into another room.



(a) What is the average number of times he goes to room 2 if it started in room 2? To room 3?

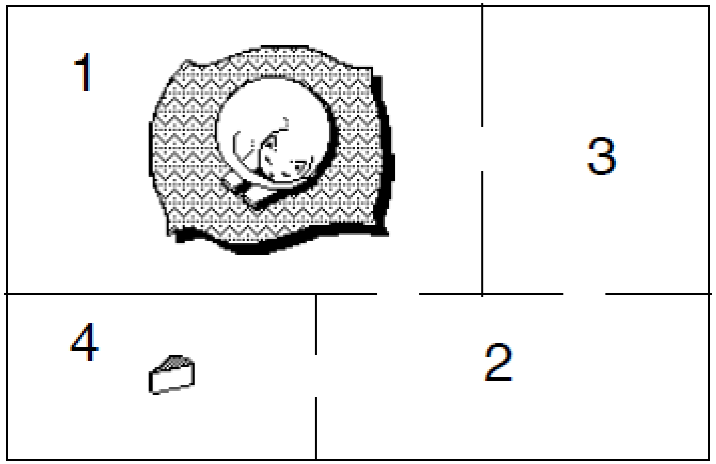
(b) What is the average number of steps he move before to room 1 or 4 if he started in room 3?

NR Matrix

- The <u>*NR* matrix</u> is the product of the fundamental (*N*) matrix and the *R* matrix.
- It gives the probabilities of eventually moving from each nonabsorbing state to each absorbing state.
- Multiplying any vector of initial nonabsorbing state probabilities by *NR* gives the vector of probabilities for the process eventually reaching each of the absorbing states. Such computations enable economic analyses of systems and policies.

Example

A rat is placed in the maze of the figure. In room 1 is a cat and in room 4 is a piece of cheese. If the rat enters either of these rooms he does not leave it. If he is in one of the other rooms, each time period he chooses at random one of the doors of the room he is in and moves into another room.



(a) If the rat starts out in Room 2, what is the probability he winds up in room 1? Room 4?

Quick Review Question

Follow Up Assignment

Summary of Main Teaching Points

Question and Answer Session

Q & A

Next Lesson

Queuing System