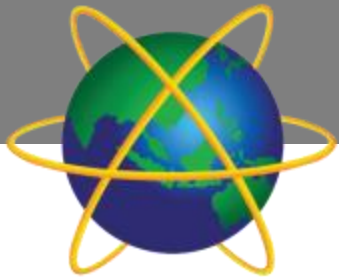


Operational Research and Optimisation

AQ052-3-M-ORO and VD1

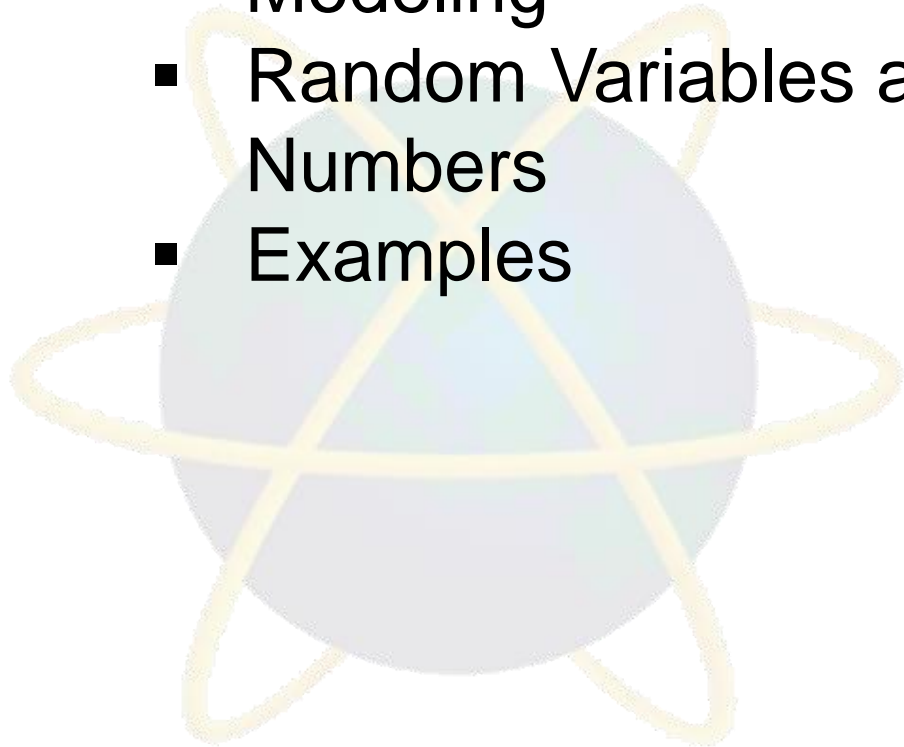


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Simulation

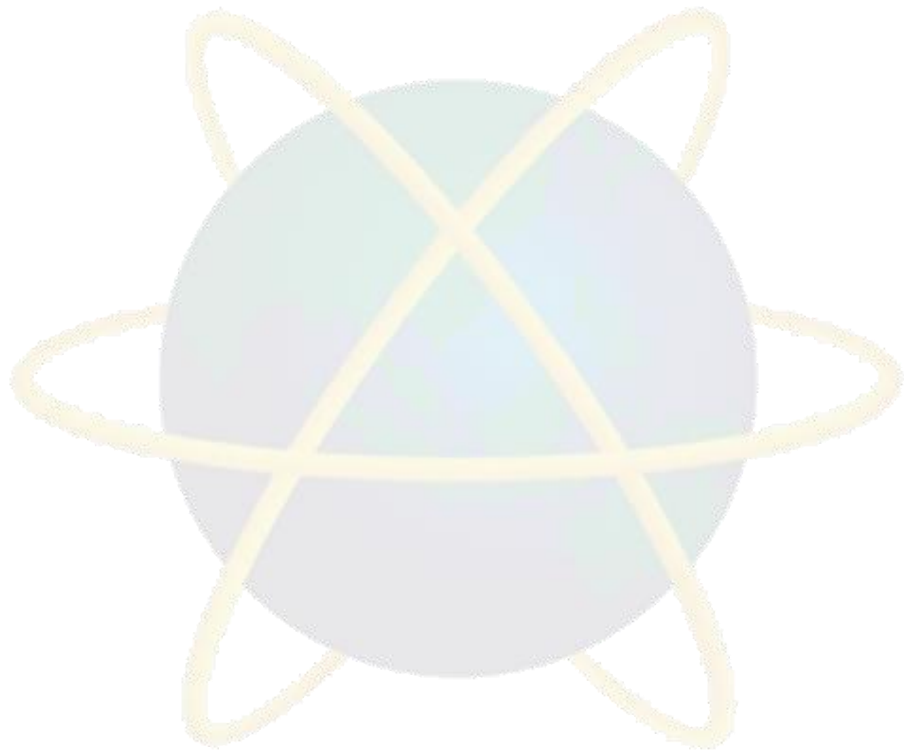
Topic & Structure of the lesson

- Advantages and Disadvantages of Using Simulation
- Modeling
- Random Variables and Pseudo-Random Numbers
- Examples



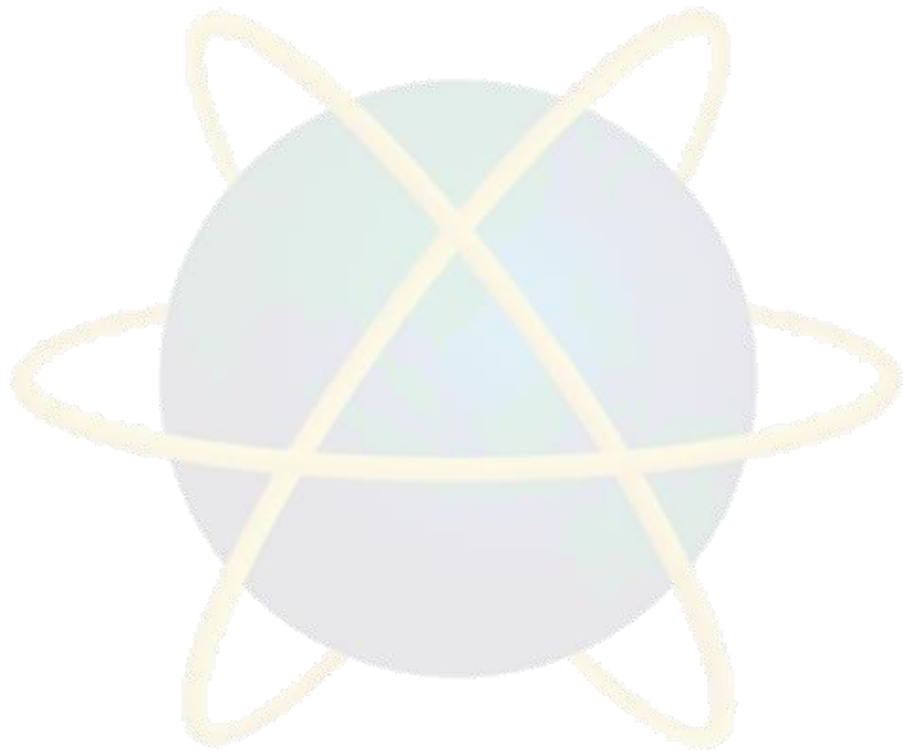
Learning Outcomes

- At the end of this topic, You should be able to use Monte Carlo simulation to model a complex real world problem.



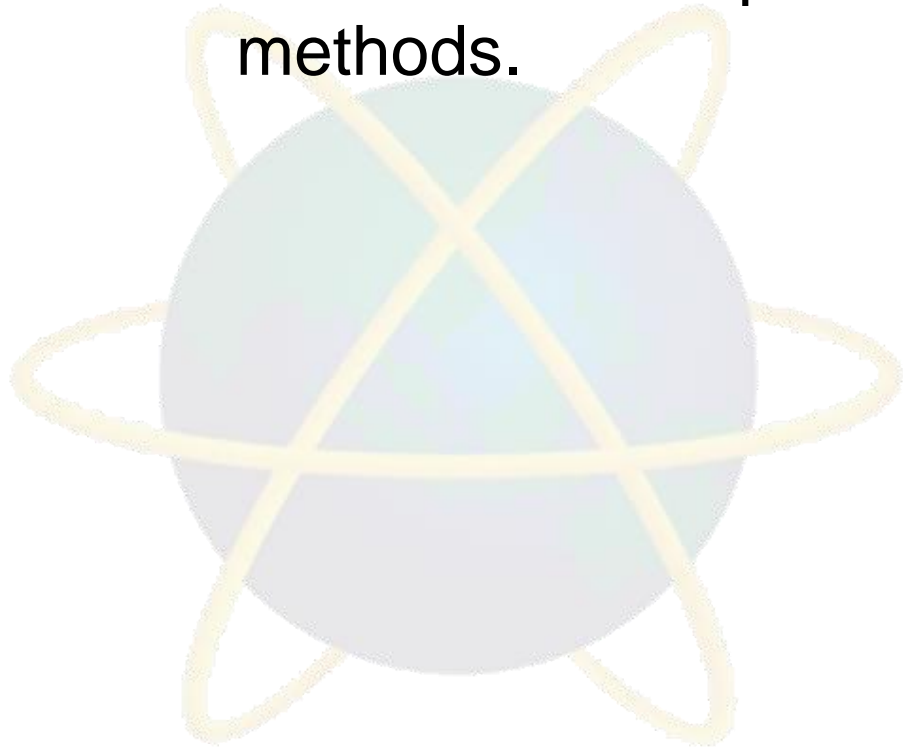
Key Terms you must be able to use

If you have mastered this topic, you should be able to use the following terms correctly in your assignments and exams:



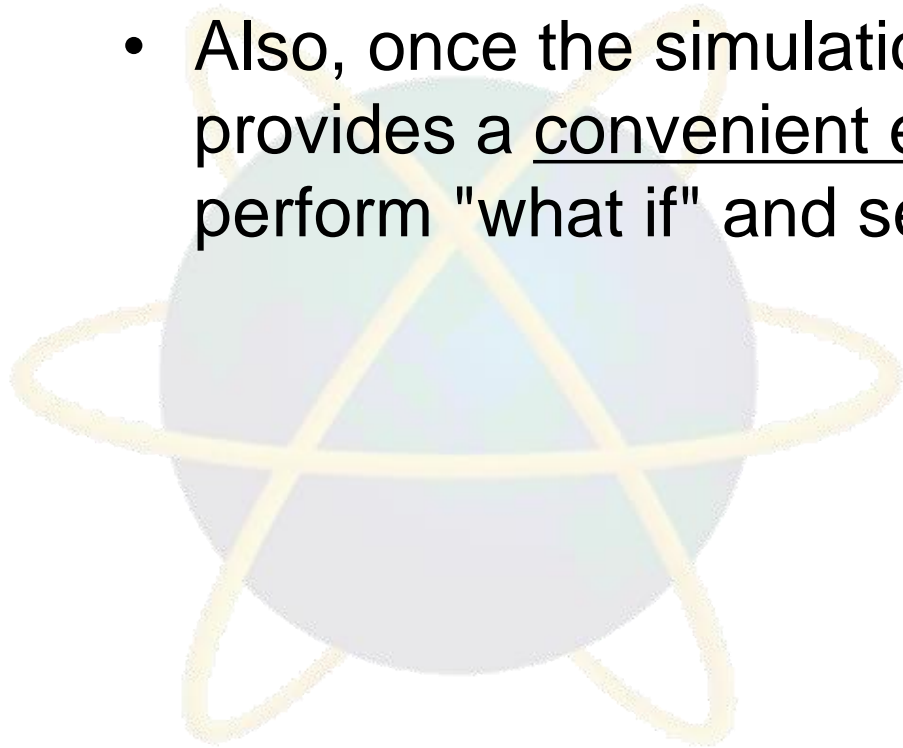
Simulation

- Simulation is one of the most frequently employed management science techniques.
- It is typically used to model random processes that are too complex to be solved by analytical methods.



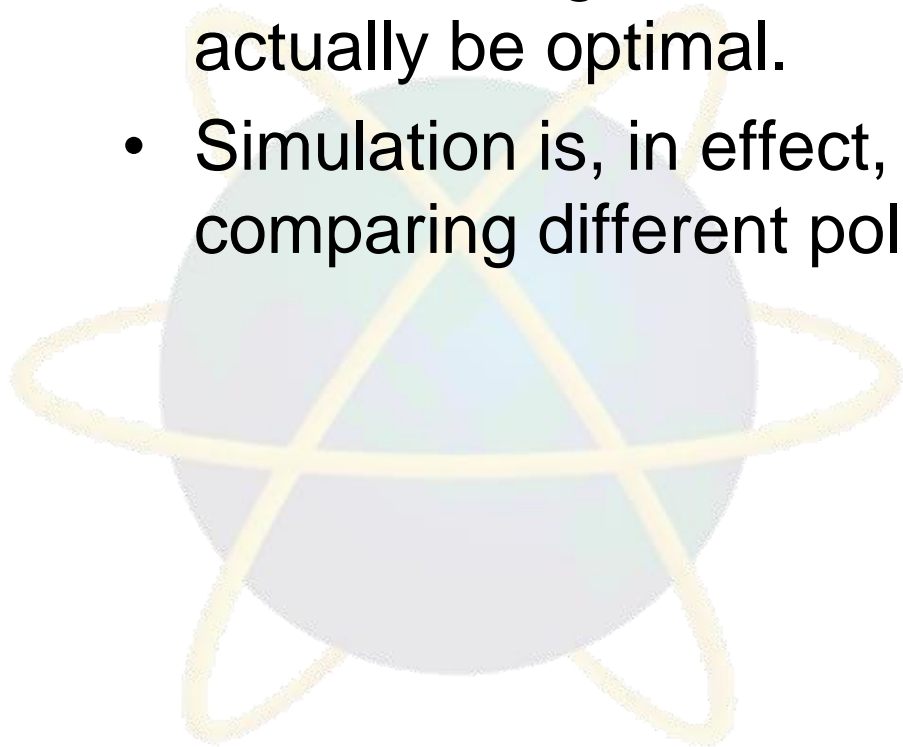
Advantages of Simulation

- Among the advantages of simulation is the ability to gain insights into the model solution which may be impossible to attain through other techniques.
- Also, once the simulation has been developed, it provides a convenient experimental laboratory to perform "what if" and sensitivity analysis.



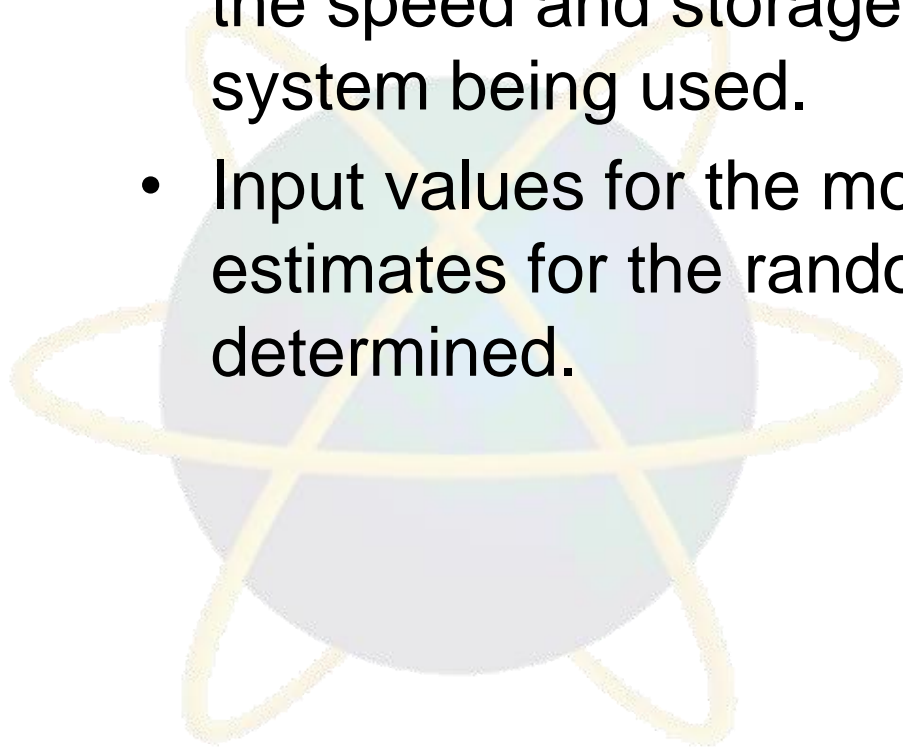
Disadvantages of Simulation

- A large amount of time may be required to develop the simulation.
- There is no guarantee that the solution obtained will actually be optimal.
- Simulation is, in effect, a trial and error method of comparing different policy inputs.



Simulation Modeling

- One begins a simulation by developing a mathematical statement of the problem.
- The model should be realistic yet solvable within the speed and storage constraints of the computer system being used.
- Input values for the model as well as probability estimates for the random variables must then be determined.



Random Numbers and Simulating Values of Random Variables

- Random variable values are utilized in the model through a technique known as Monte Carlo simulation.
- Each random variable is mapped to a set of numbers so that each time one number in that set is generated (eg. random number), the corresponding value of the random variable is given as an input to the model.
- The mapping is done in such a way that the likelihood that a particular number is chosen is the same as the probability that the corresponding value of the random variable occurs.

Example: Dynogen, Inc.



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The price change of shares of Dynogen, Inc. has been observed over the past 50 trades. The frequency distribution is as follows:

<u>Price Change</u>	<u>Number of Trades</u>
-3/8	4
-1/4	2
-1/8	8
0	20
+1/8	10
+1/4	3
+3/8	2
+1/2	<u>1</u>
Total = 50	

Example: Dynogen, Inc.

- Relative Frequency Distribution and Random Number Mapping

<u>Price Change</u>	<u>Relative Frequency</u>	<u>Random Numbers</u>
-3/8	.08	00 - 07
-1/4	.04	08 - 11
-1/8	.16	12 - 27
0	.40	28 - 67
+1/8	.20	68 - 87
+1/4	.06	88 - 93
+3/8	.04	94 - 97
+1/2	<u>.02</u>	98 - 99
TOTAL	1.00	

Example: Dynogen, Inc.

If the current price per share of Dynogen is 23, use random numbers to simulate the price per share over the next 10 trades. Use the following stream of random numbers:

21, 84, 07, 30, 94, 57, 57, 19, 84, 84

Example: Dynogen, Inc.

- Simulation Worksheet

<u>Trade Number</u>	<u>Random Number</u>	<u>Price Change</u>	<u>Stock Price</u>
1	21	-1/8	22 7/8
2	84	+1/8	23
3	07	-3/8	22 5/8
4	30	0	22 5/8
5	94	+3/8	23
6	57	0	23
7	57	0	23
8	19	-1/8	22 7/8
9	84	+1/8	23
10	84	+1/8	23 1/8

Example: Dynogen, Inc.

	A	B	C	D	E	F
1	Lower	Upper		Trade	Price	Stock
2	Random	Random	Price	Number	Change	Price
3	Number	Number	Change	1	0.125	23.125
4	0.00	0.08	-0.375	2	0.375	23.500
5	0.08	0.12	-0.250	3	0.000	23.500
6	0.12	0.28	-0.125	4	0.000	23.500
7	0.28	0.68	0.000	5	0.000	23.500
8	0.68	0.88	0.125	6	0.000	23.500
9	0.88	0.94	0.250	7	0.125	23.625
10	0.94	0.98	0.375	8	0.125	23.750
11	0.98	1.00	0.500	9	0.000	23.750
12				10	0.125	23.875

Example: Wayne International Airport

Wayne International Airport primarily serves domestic air traffic. Occasionally, however, a chartered plane from abroad will arrive with passengers bound for Wayne's two great amusement parks, Algorithmland and Giffith's Cherry Preserve.

Whenever an international plane arrives at the airport the two customs inspectors on duty set up operations to process the passengers.

Incoming passengers must first have their passports and visas checked. This is handled by one inspector. The time required to check a passenger's passports and visas can be described by the probability distribution on the next slide.

Example: Wayne International Airport

<u>Time Required to Check a Passenger's Passport and Visa</u>	<u>Probability</u>
20 seconds	.20
40 seconds	.40
60 seconds	.30
80 seconds	.10

Example: Wayne International Airport

After having their passports and visas checked, the passengers next proceed to the second customs official who does baggage inspections. Passengers form a single waiting line with the official inspecting baggage on a first come, first served basis. The time required for baggage inspection has the following probability distribution:

<u>Time Required For Baggage Inspection</u>	<u>Probability</u>
No Time	.25
1 minute	.60
2 minutes	.10
3 minutes	.05

Example: Wayne International Airport

- Random Number Mapping

<u>Time Required to Check a Passenger's Passport and Visa</u>	<u>Probability</u>	<u>Random Numbers</u>
20 seconds	.20	00 - 19
40 seconds	.40	20 - 59
60 seconds	.30	60 - 89
80 seconds	.10	90 - 99

Example: Wayne International Airport

- Random Number Mapping

<u>Time Required For Baggage Inspection</u>	<u>Probability</u>	<u>Random Numbers</u>
No Time	.25	00 - 24
1 minute	.60	25 - 84
2 minutes	.10	85 - 94
3 minutes	.05	95 - 99

Example: Wayne International Airport

- Next-Event Simulation Records

For each passenger the following information must be recorded:

- When his service begins at the passport control inspection
- The length of time for this service
- When his service begins at the baggage inspection
- The length of time for this service

Example: Wayne International Airport

- Time Relationships

Time a passenger begins service
by the passport inspector

= (Time the previous passenger started passport service) + (Time of previous passenger's passport service)

Example: Wayne International Airport

- Time Relationships

Time a passenger begins service
by the baggage inspector

(If passenger does not wait in line for baggage inspection)
= (Time passenger completes service
with the passport control inspector)

(If the passenger does wait in line for baggage inspection)
= (Time previous passenger completes
service with the baggage inspector)

Example: Wayne International Airport

- Time Relationships

Time a customer completes service
at the baggage inspector

= (Time customer begins service with baggage inspector) + (Time required for baggage inspection)

Example: Wayne International Airport

A chartered plane from abroad lands at Wayne Airport with 80 passengers. Simulate the processing of the first 10 passengers through customs. Use the following random numbers:

For passport control:

93, 63, 26, 16, 21, 26, 70, 55, 72, 89

For baggage inspection:

13, 08, 60, 13, 68, 40, 40, 27, 23, 64

Example: Wayne International Airport

- Simulation Worksheet (partial)

Pass. Num	<u>Passport Control</u>				<u>Baggage Inspections</u>			
	Time Begin	Rand. Num.	Service Time	Time End	Time Begin	Rand. Num.	Service Time	Time End
1	0:00	93	1:20	1:20	1:20	13	0:00	1:20
2	1:20	63	1:00	2:20	2:20	08	0:00	2:20
3	2:20	26	:40	3:00	3:00	60	1:00	4:00
4	3:00	16	:20	3:20	4:00	13	0:00	4:00
5	3:20	21	:40	4:00	4:00	68	1:00	5:00

Example: Wayne International Airport

- Simulation Worksheet (continued)

Pass. Num	<u>Passport Control</u>				<u>Baggage Inspections</u>			
	Time Begin	Rand. Num.	Service Time	Time End	Time Begin	Rand. Num.	Service Time	Time End
6	4:00	26	:40	4:40	5:00	40	1:00	6:00
7	4:40	70	1:00	5:40	6:00	40	1:00	7:00
8	5:40	55	:40	6:20	7:00	27	1:00	8:00
9	6:20	72	1:00	7:20	8:00	23	0:00	8:00
10	7:20	89	1:00	8:20	8:20	64	1:00	9:20

Example: Wayne International Airport

- Explanation

For example, passenger 1 begins being served by the passport control inspector immediately. His service time is 1:20 (80 seconds) at which time he goes immediately to the baggage inspector who waves him through without inspection.

Passenger 2 begins service with passport inspector 1:20 minutes (80 seconds) after arriving there (as this is when passenger 1 is finished) and requires 1:00 minute (60 seconds) for passport inspection. He is waved through baggage inspection as well.

This process continues in this manner.

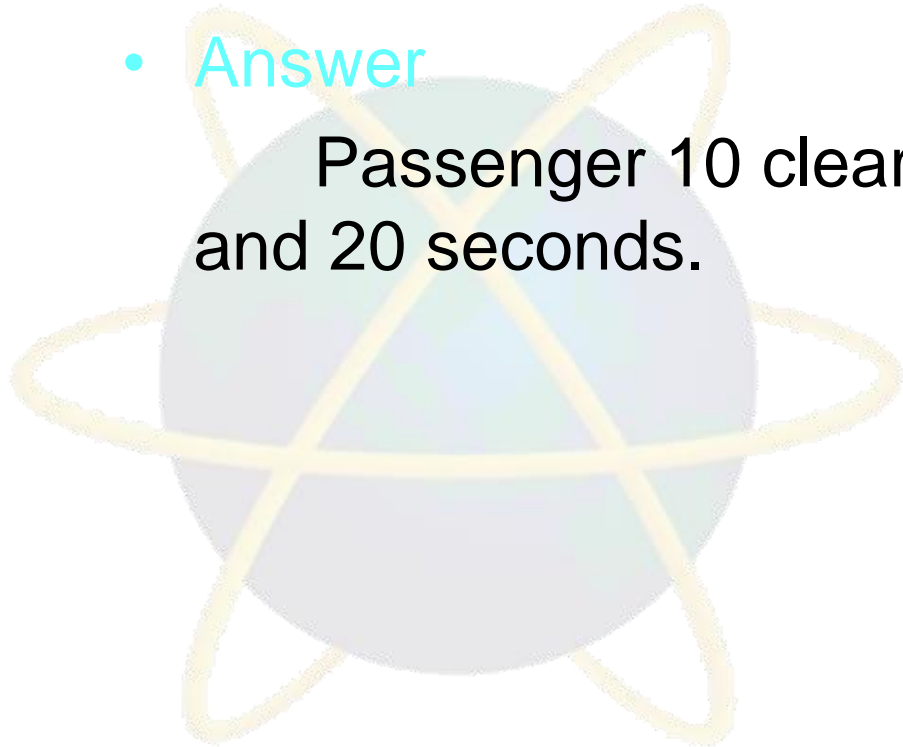
Example: Wayne International Airport

- Question

How long will it take for the first 10 passengers to clear customs?

- Answer

Passenger 10 clears customs after 9 minutes and 20 seconds.



Example: Wayne International Airport

- Question

What is the average length of time a customer waits before having his bags inspected after he clears passport control?



Example: Wayne International Airport

- Answer

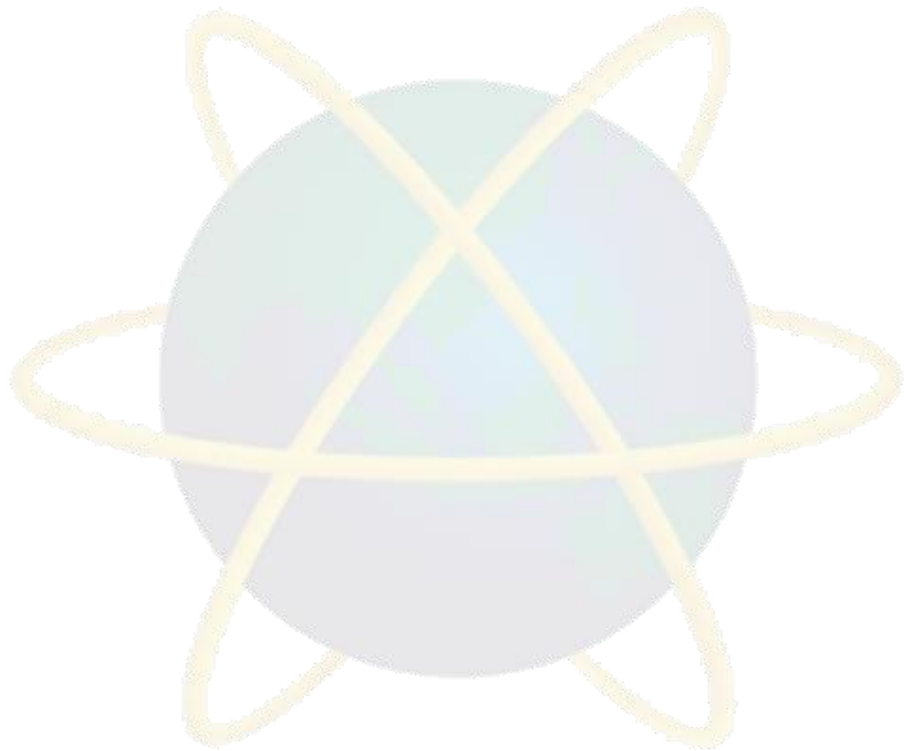
For each passenger calculate his waiting time:

(Baggage Inspection Begins) - (Passport Control Ends)

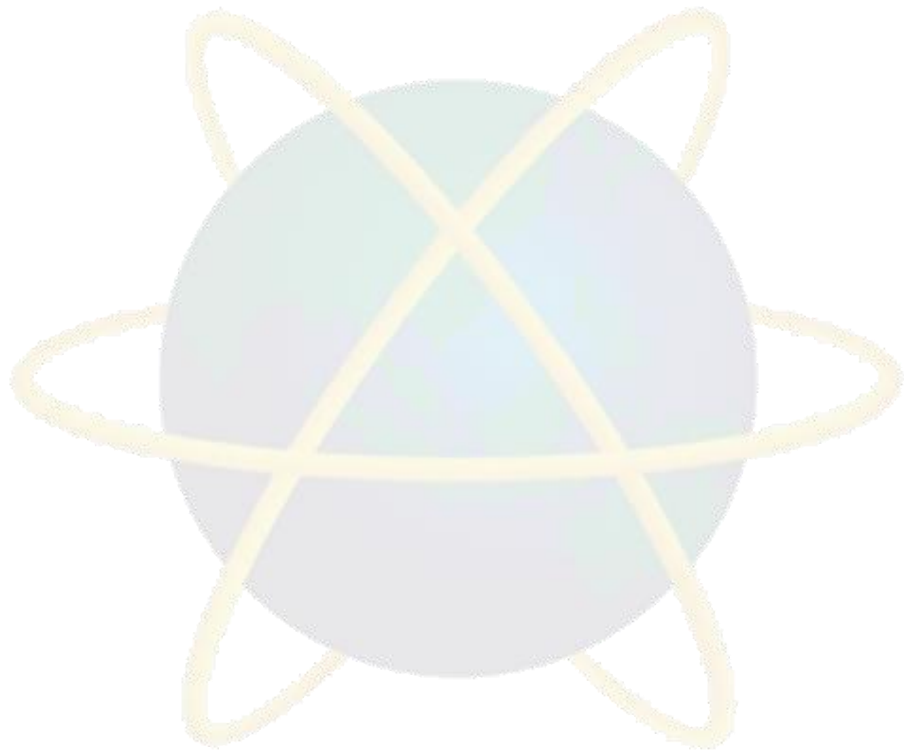
$$= 0+0+0+40+0+20+20+40+40+0 = 160 \text{ seconds.}$$

$$160/10 = 16 \text{ seconds per passenger}$$

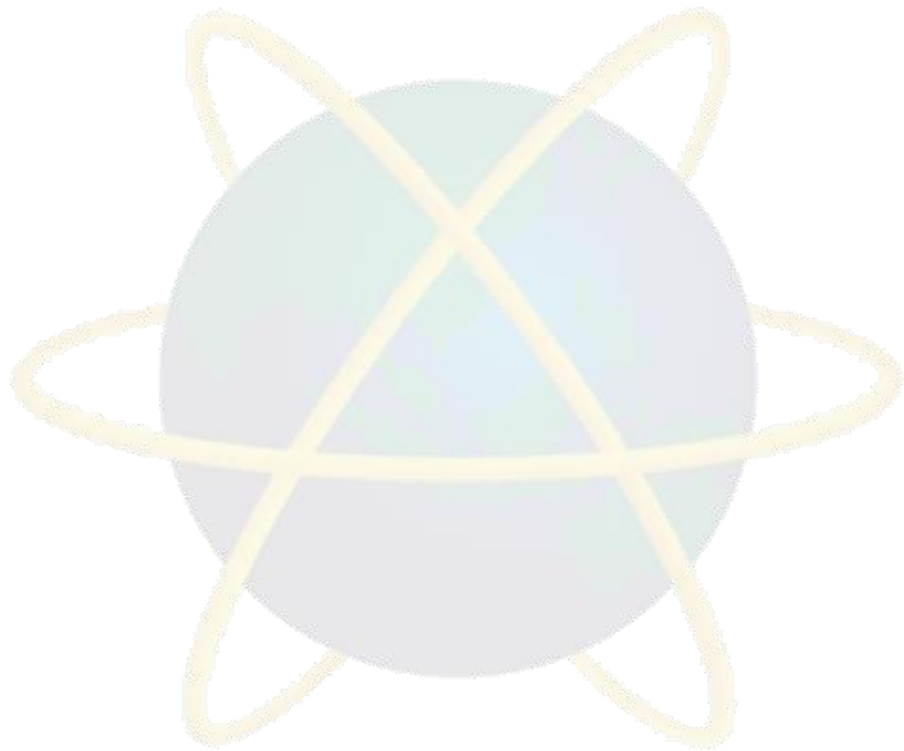
Quick Review Question



Follow Up Assignment

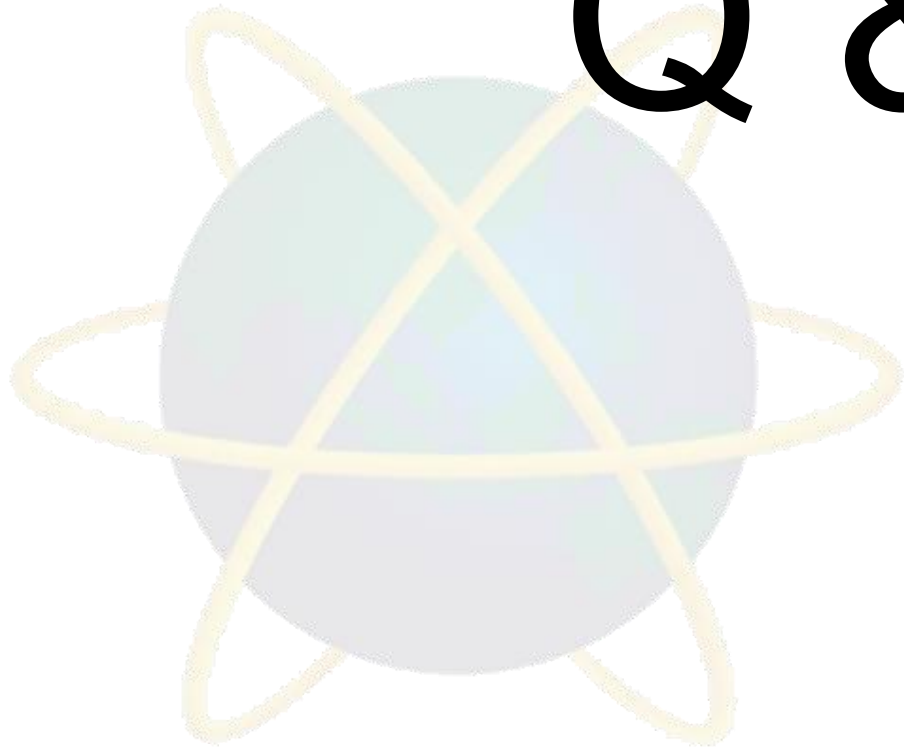


Summary of Main Teaching Points



Question and Answer Session

Q & A



Next Lesson

Project Scheduling

