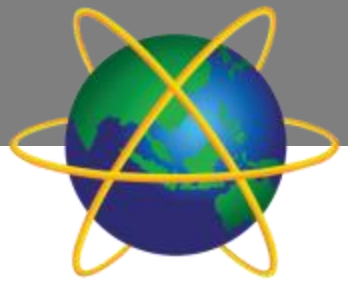
Operational Research and Optimisation

AQ052-3-M-ORO and VD1



Goal Programming

Topic & Structure of the lesson

- A goal programming model formulations
- Goal programming algorithms
- The preemptive method

Learning Outcomes

- **A the end of this topic, You should be able to model a real world problem into a goal programming model with different goals and priority levels.**

Key Terms you must be able to use

If you have mastered this topic, you should be able to use the following terms correctly in your assignments and exams:

What is Goal Programming?

- Single objective in formulating LPs so far
  - Max profit
  - Min cost
- Several simultaneous objectives in practice
  - Max profit
  - Increase reliability
  - Improve employee morale
  - Improve company's image
- Goal programming: a way of solving multi-objective problems

Categorization of Goal Programming

- **Preemptive goal programming**
  - The goals are listed in the order of their importance.
  - It begins by focusing solely on the most important goal
  - It next does the same for the second most important goal (as is possible without hurting the first goal)
  - It continues with the following goals (as is possible without hurting the previous more important goals)

Goal Programming Example Problem Data

Beaver Creek Pottery Company Example:

Maximize $Z = \$40x_1 + 50x_2$

subject to:

$1x_1 + 2x_2 \leq 40$ hours of labor

$4x_1 + 3x_2 \leq 120$ pounds of clay

$x_1, x_2 \geq 0$

Where: x_1 = number of bowls produced

x_2 = number of mugs produced

Goal Programming Example Problem Data

- Adding objectives (goals) ***in order of importance***, the company*:*
  1. Does not want to use fewer than 40 hours of labor per day.
  2. Would like to achieve a satisfactory profit level of $1,600 per day.
  3. Prefers not to keep more than 120 pounds of clay on hand each day.
  4. Would like to minimize the amount of overtime.

Formulation of GP Problems

Deviations: the amount away from the desired standards or objectives:

- Overachievement ($d^+_i \geq 0$) vs. Underachievement ($d^-_i \geq 0$)

- Desirable vs. Undesirable Deviations: (depend on the objectives)
  - Max goals (≥) - the more the better - d^+_i desirable.
  - Min goals (≤) - the less the better - d^-_i desirable.
  - Exact goals (=) - exactly equal - both d^+_i and d^-_i undesirable

- In GP, the objective is to minimize the (weighted) sum of undesirable deviations (all undesirable d^+_i and $d^-_i \rightarrow\rightarrow 0$).

- For each goal, at least, one of d^+_i and d^-_i must be equal to "0"

Goal Programming Model Formulation
Goal Constraints

Labor goal:

$$x_1 + 2x_2 + d_1^- - d_1^+ = 40 \quad \text{(hours/day)}$$

Profit goal:

$$40x_1 + 50x_2 + d_2^- - d_2^+ = 1{,}600 \quad \text{(\$/day)}$$

Material goal:

$$4x_1 + 3x_2 + d_3^- - d_3^+ = 120 \quad \text{(lbs of clay/day)}$$

Goal Programming Model Formulation Objective Function

1. Labor goals constraint

(priority 1 - less than 40 hours labor; priority 4 - minimum overtime):

- Minimize $P_1d_1^-$, $P_4d_1^+$

2. Add profit goal constraint

(priority 2 - achieve profit of $1,600):

- Minimize $P_1d_1^-$, $P_2d_2^-$, $P_4d_1^+$

3. Add material goal constraint

(priority 3 - avoid keeping more than 120 pounds of clay on hand):

- Minimize $P_1d_1^-$, $P_2d_2^-$, $P_3d_3^+$, $P_4d_1^+$

Goal Programming Model Formulation Complete Model

Complete Goal Programming Model:

Minimize $P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_1^+$

subject to:

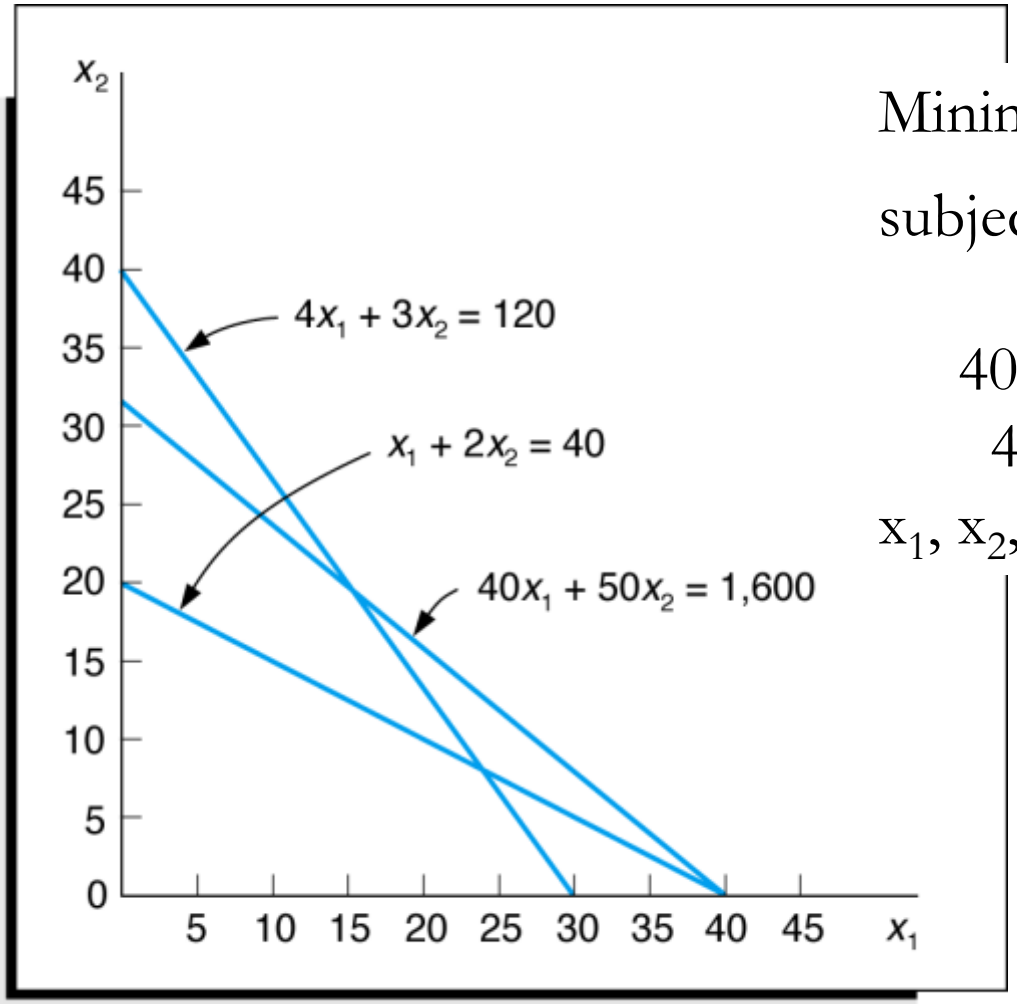
$$x_1 + 2x_2 + d_1^- - d_1^+ = 40 \quad \text{(labor)}$$

$$40x_1 + 50x_2 + d_2^- - d_2^+ = 1{,}600 \quad \text{(profit)}$$

$$4x_1 + 3x_2 + d_3^- - d_3^+ = 120 \quad \text{(clay)}$$

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

Goal Programming
Graphical Interpretation (1 of 6)

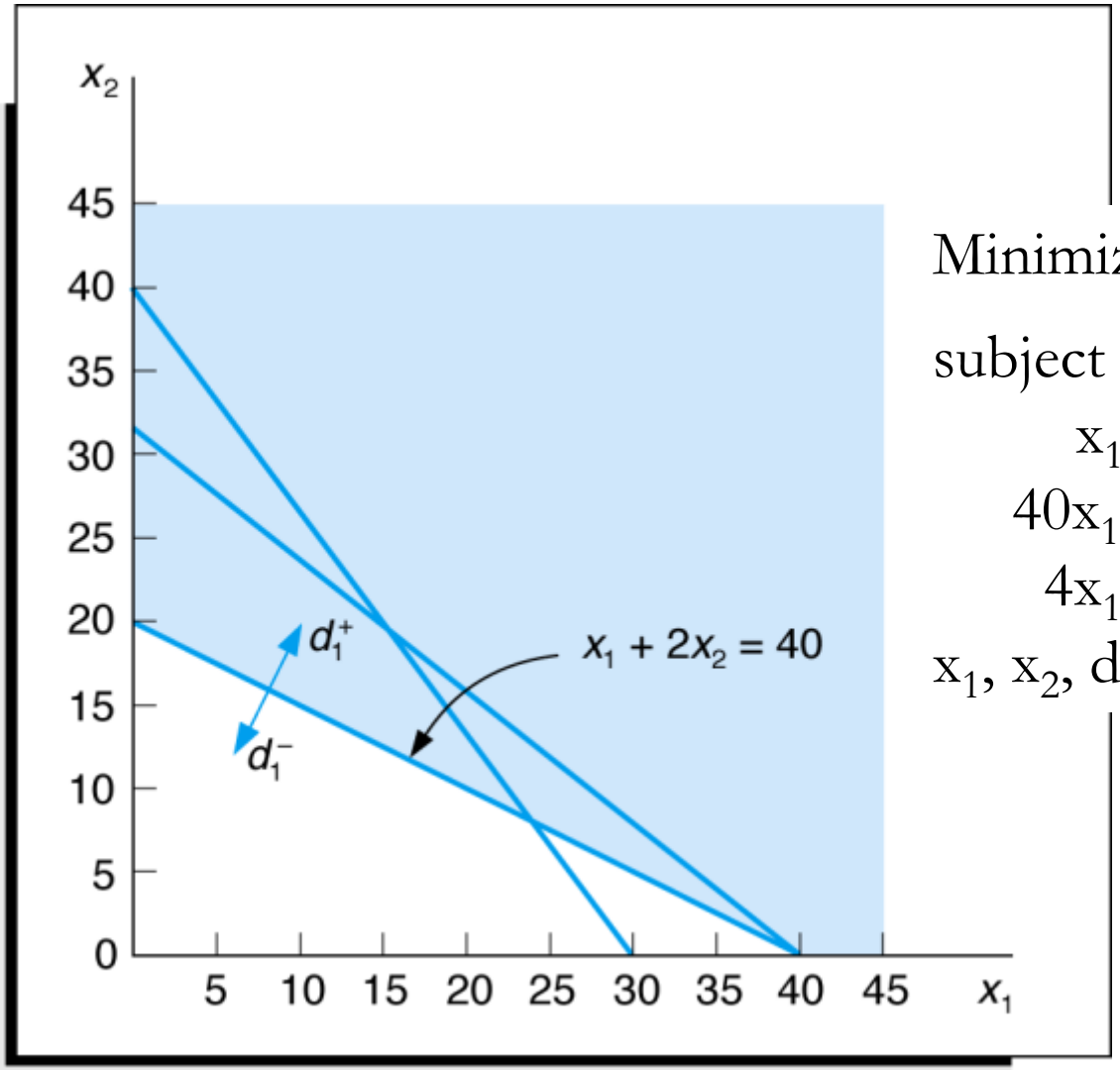


Minimize $P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_1^+$

subject to:

$$x_1 + 2x_2 + d_1^- - d_1^+ = 40$$
$$40x_1 + 50\,x_2 + d_2^- - d_2^+ = 1{,}600$$
$$4x_1 + 3x_2 + d_3^- - d_3^+ = 120$$
$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

Goal Programming
Graphical Interpretation (2 of 6)



Minimize $P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_1^+$

subject to:

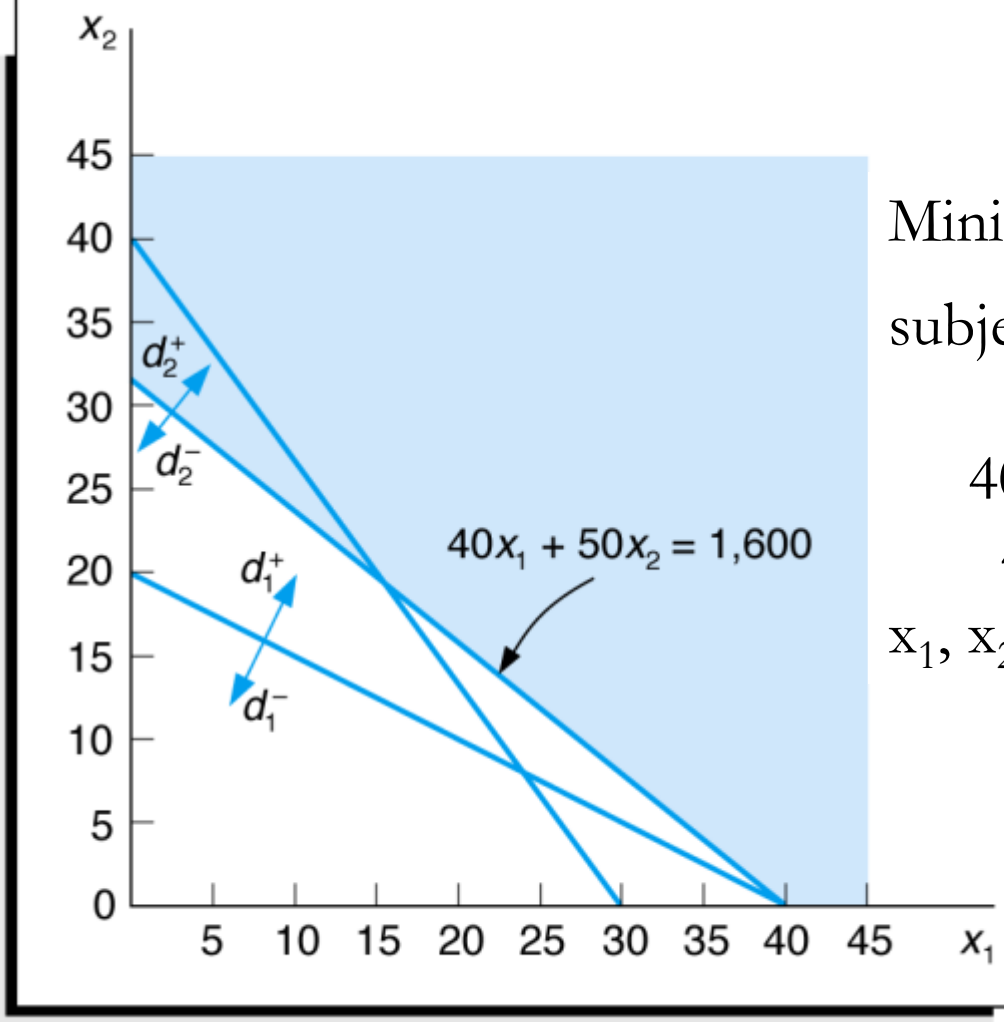
$$x_1 + 2x_2 + d_1^- - d_1^+ = 40$$

$$40x_1 + 50x_2 + d_2^- - d_2^+ = 1{,}600$$

$$4x_1 + 3x_2 + d_3^- - d_3^+ = 120$$

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

Goal Programming
Graphical Interpretation (3 of 6)



Minimize $P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_1^+$

subject to:

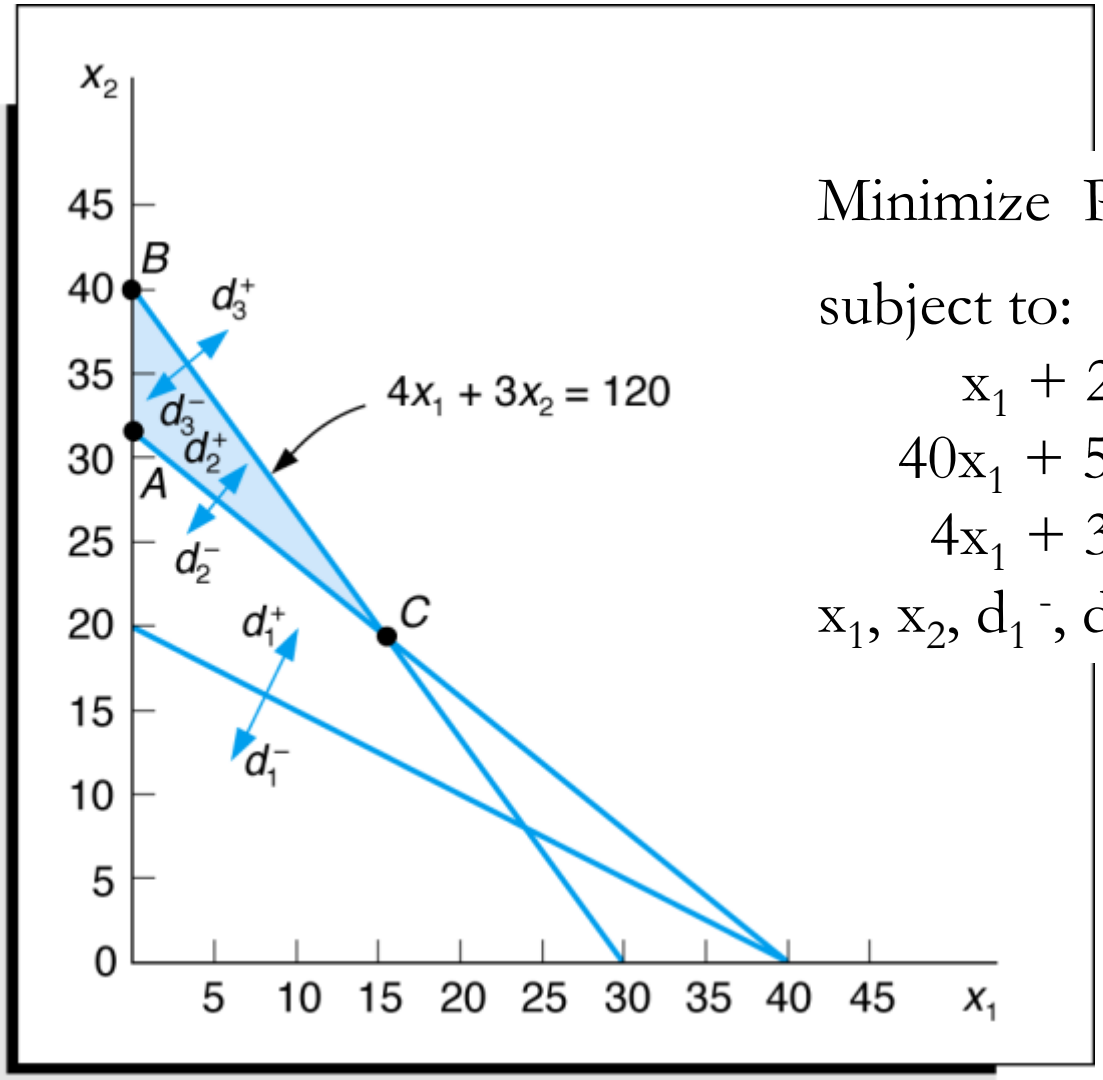
$$x_1 + 2x_2 + d_1^- - d_1^+ = 40$$

$$40x_1 + 50x_2 + d_2^- - d_2^+ = 1,600$$

$$4x_1 + 3x_2 + d_3^- - d_3^+ = 120$$

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

Goal Programming
Graphical Interpretation (4 of 6)



Minimize $P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_1^+$

subject to:

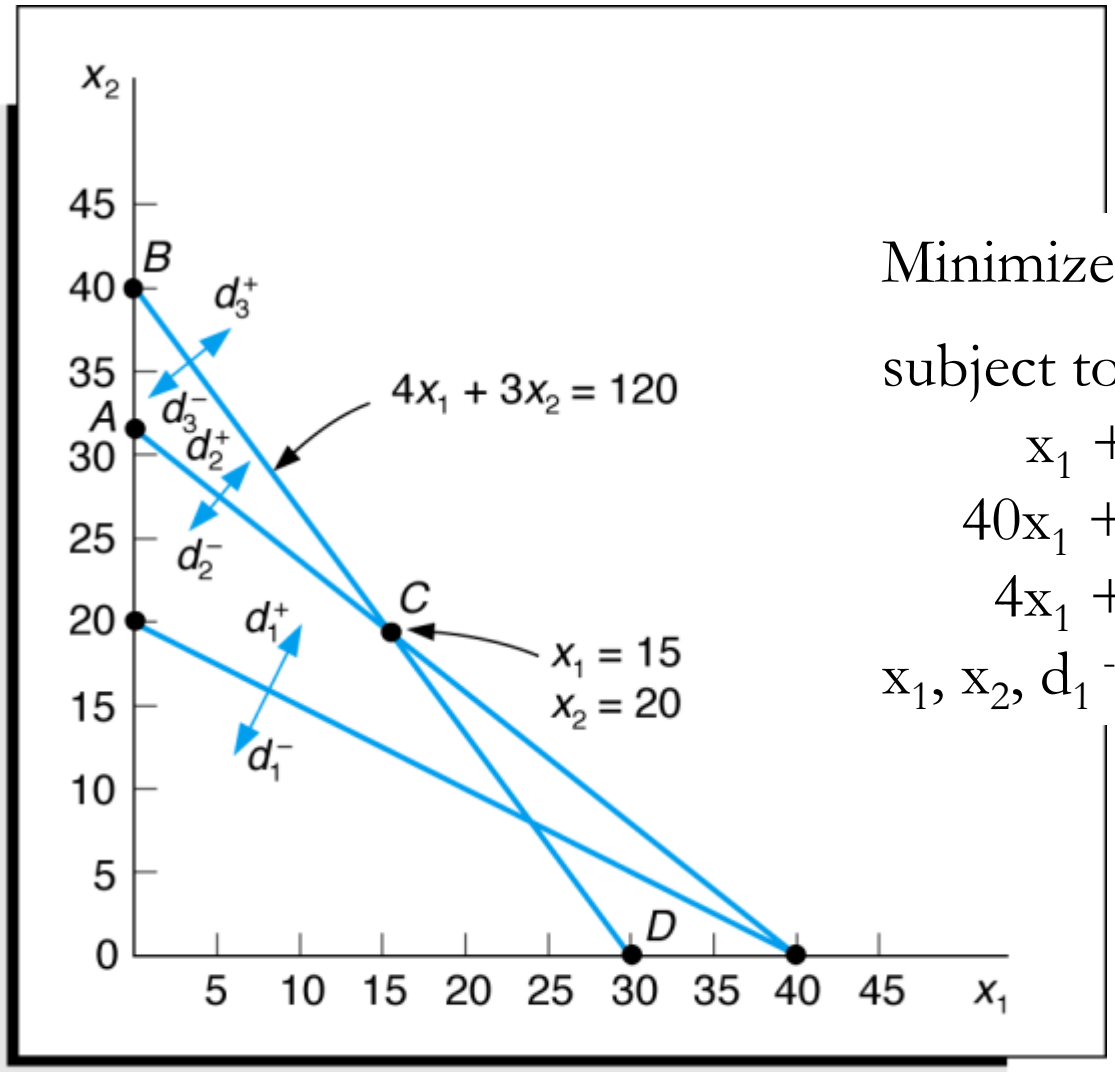
$$x_1 + 2x_2 + d_1^- - d_1^+ = 40$$

$$40x_1 + 50\,x_2 + d_2^- - d_2^+ = 1{,}600$$

$$4x_1 + 3x_2 + d_3^- - d_3^+ = 120$$

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

Goal Programming
Graphical Interpretation (5 of 6)



Minimize $P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_1^+$

subject to:

$$x_1 + 2x_2 + d_1^- - d_1^+ = 40$$
$$40x_1 + 50x_2 + d_2^- - d_2^+ = 1{,}600$$
$$4x_1 + 3x_2 + d_3^- - d_3^+ = 120$$
$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

Goal Programming
Graphical Interpretation (6 of 6)

Goal programming solutions do not always achieve all goals and they ***are not "optimal"***, they achieve the best or most satisfactory solution possible.

Minimize $P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_1^+$

subject to:

$$x_1 + 2x_2 + d_1^- - d_1^+ = 40$$
$$40x_1 + 50x_2 + d_2^- - d_2^+ = 1{,}600$$
$$4x_1 + 3x_2 + d_3^- - d_3^+ = 120$$
$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

Solution: $x_1 = 15$ bowls

$x_2 = 20$ mugs

$d_1^+ = 15$ hours

Example

	Unit Contribution of Product		
	1	2	3
Total profit (millions of dollars)	12	9	15
Employment level (hundreds of employees)	5	3	4
Capital investment (millions of dollars)	5	7	8

The goals in the order of importance are:

1. Achieve a total profit (net present value) of at least $125 million.
2. Avoid decreasing the employment level below 4000 employees.
3. Hold the capital investment down to no more than $55 million.
4. Avoid increasing the employment level above 4000 employees.

Solve the problem above to determine number of Products 1, 2 and 3 to be produced.

Example

A new advertising agency with 10 employees, has received a contract to promote a new product. The agency can advertise using radio and television. The following table gives the number of people reached by each type of advertisement and the cost and labor requirements.

	Radio	Television
Exposure (in millions of persons)/min	4	8
Cost (in S'000)/min	8	24
Assigned employees/min	1	2

- The contract prohibits the company from using more than 6 minutes of radio advertisement.
- Radio and television advertisements need to reach at least 45 million people.
- The company has a budget goal of $100,000 for the project.

How many minutes of radio and television advertisement should the company use? Assuming the exposure goal is twice as important as the budget goal .

EXERCISE

Public Relation Company has contracted to do a survey following an election primary in Malaysia. The firm must assign interviews to carry out the survey. The interviews are conducted by telephone and in person. One person can conduct 80 telephone interviews or 40 personal interviews a day. It costs $50 per day for a telephone interviewer and $70 per day for a personal interviewer. The following three goals, which are listed in order their priority, have been established by the firm to ensure a representative survey:

a. At least 3000 total interviews should be conducted.
b. The firm wants to maintain its daily budget of $2500.
c. At least 1000 interviews should be by telephone.

Formulate a goal programming model to determine the number of interviewers to hire to satisfy these goals and then solve the model.

Quick Review Question

Follow Up Assignment

Summary of Main Teaching Points

Question and Answer Session

A·P·U
ASIA PACIFIC UNIVERSITY
OF TECHNOLOGY & INNOVATION

Q & A

Next Lesson

Non-Linear Programming