Operational Research and Optimisation
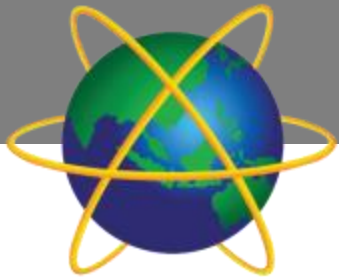
AQ052-3-M-ORO and VD1
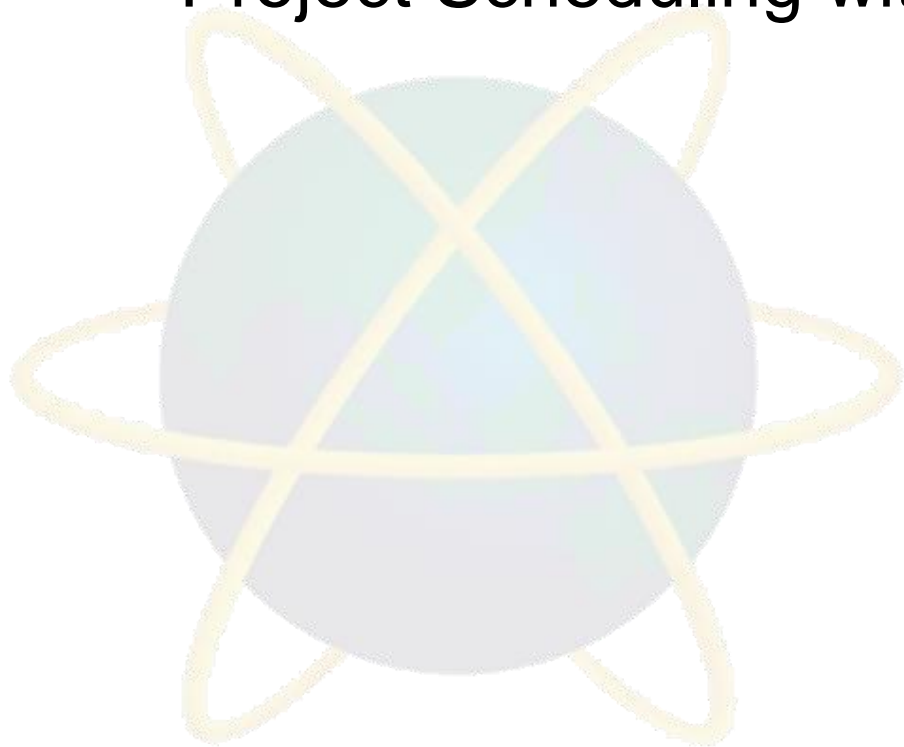
Project Scheduling

ASIA PACIFIC UNIVERSITY
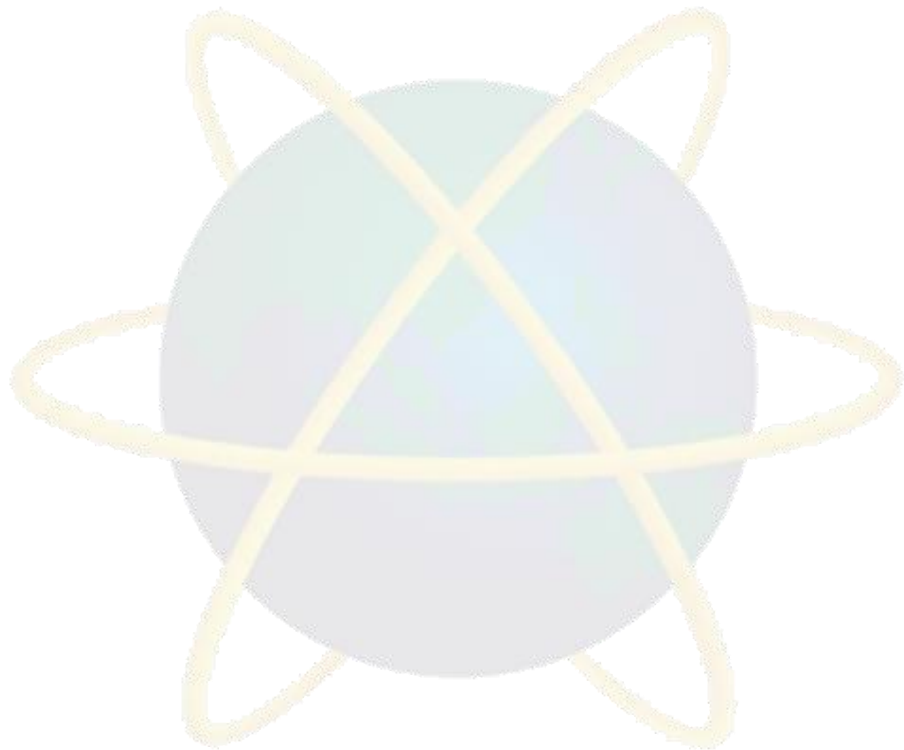OF TECHNOLOGY & INNOVATION

Topic & Structure of the lesson

- Gantt chart
- Project Scheduling with Known Activity Times
- Project Scheduling with Uncertain Activity Times
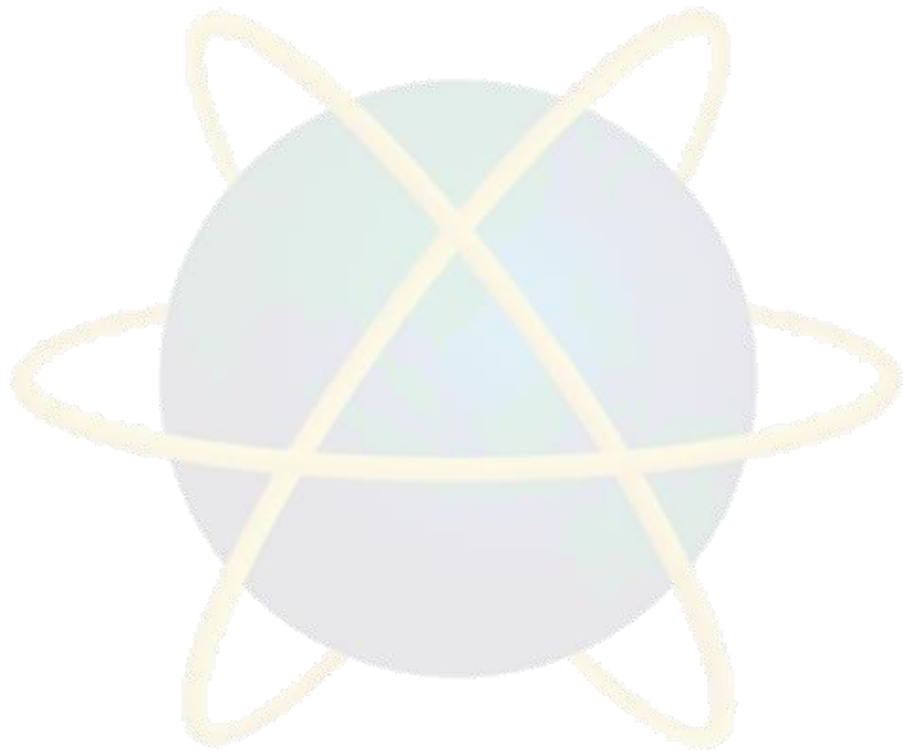
Learning Outcomes

- **A the end of this topic, You should be able to solve a project scheduling problem using PERT/CPM.**

Key Terms you must be able to use

If you have mastered this topic, you should be able to use the following terms correctly in your assignments and exams:

Gantt Chart

- Visual scheduling tool

- Graphical representation of information

- Show dependencies between tasks, personnel, and other resources allocations

- Track progress towards completion
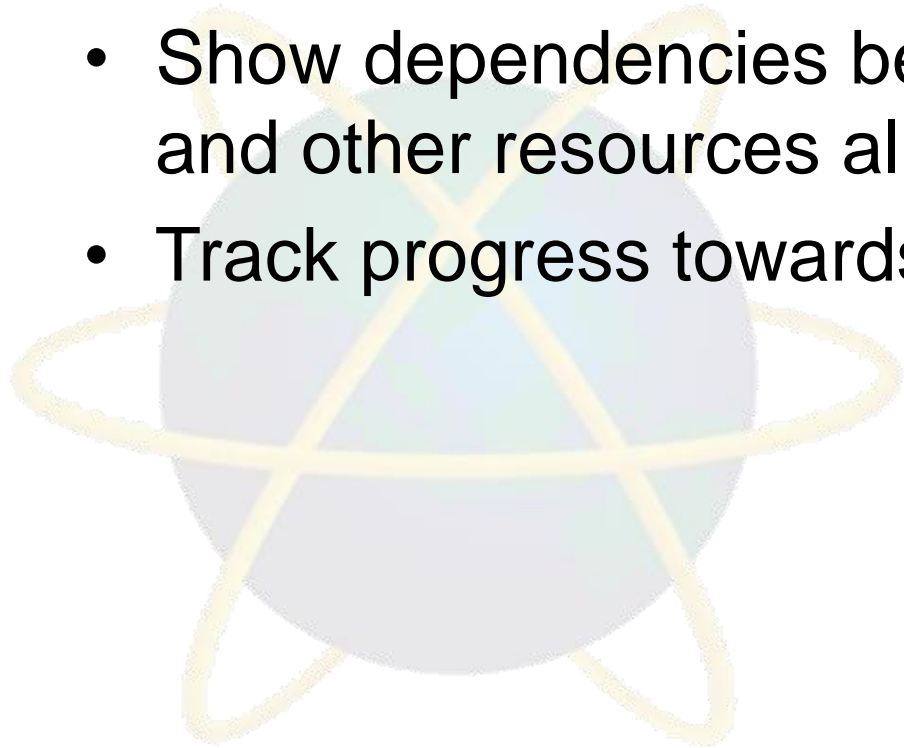
Building a Gantt Chart

- List all tasks and milestones from the project along the vertical axis

- List time frame along the horizontal axis

Activity 1

Activity 2

Milestone

Time Frame: day 1 day 2 day3
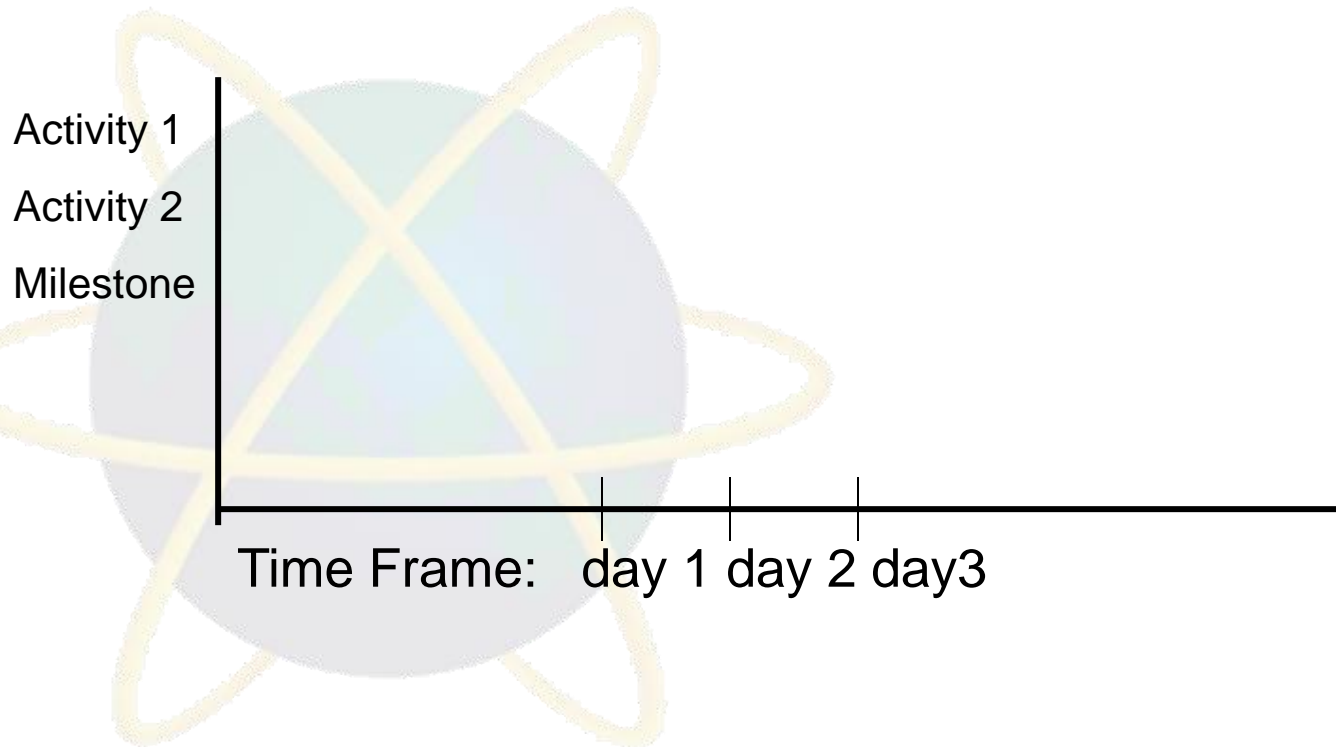
- Activities: Create box the length of each activity time duration
  - E.g., activity one is scheduled from day1-day3

Activity 1

Activity 2

Time Frame: day 1 day 2 day3
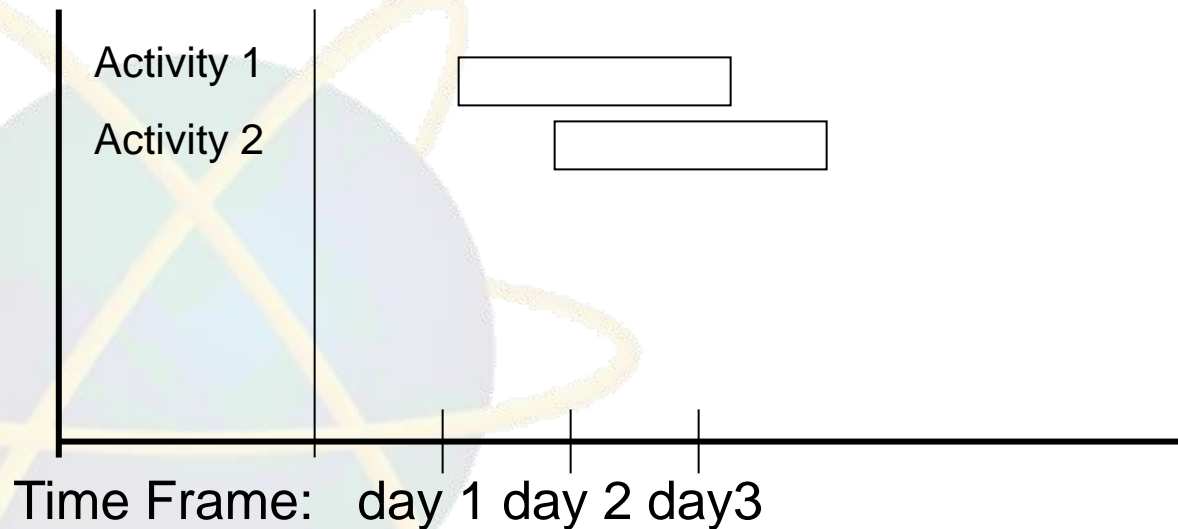
- Dependencies: Show dependencies between activities with arrows
  - E.g., activity 2 cannot start until activity 1 is complete

Activity 1

Activity 2

Time Frame: day 1 day 2 day3…
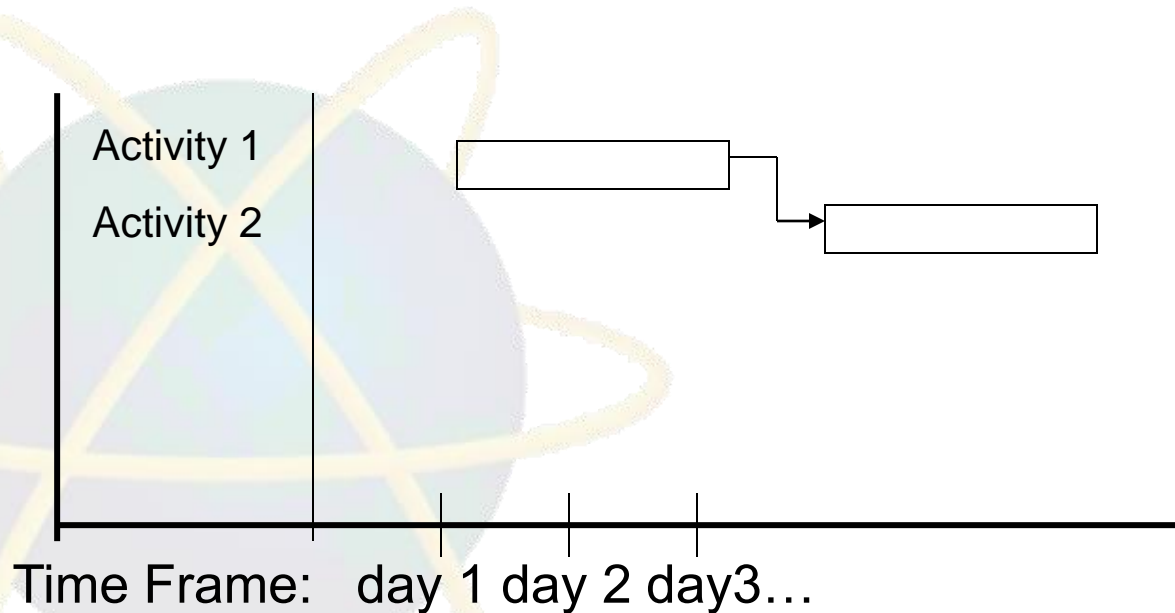
Sequence of Activities of The Project - House Building

Number	Activity	Predecessor	Duration
1	Design house and obtain financing	--	3 months
2	Lay foundation	1	2 months
3	Order and receive materials	1	1 month
4	Build house	2,3	3 months
5	Select paint	2,3	1 month
6	Select carper	5	1 month
7	Finish work	4, 6	1 month

Gantt Chart for House Building Project
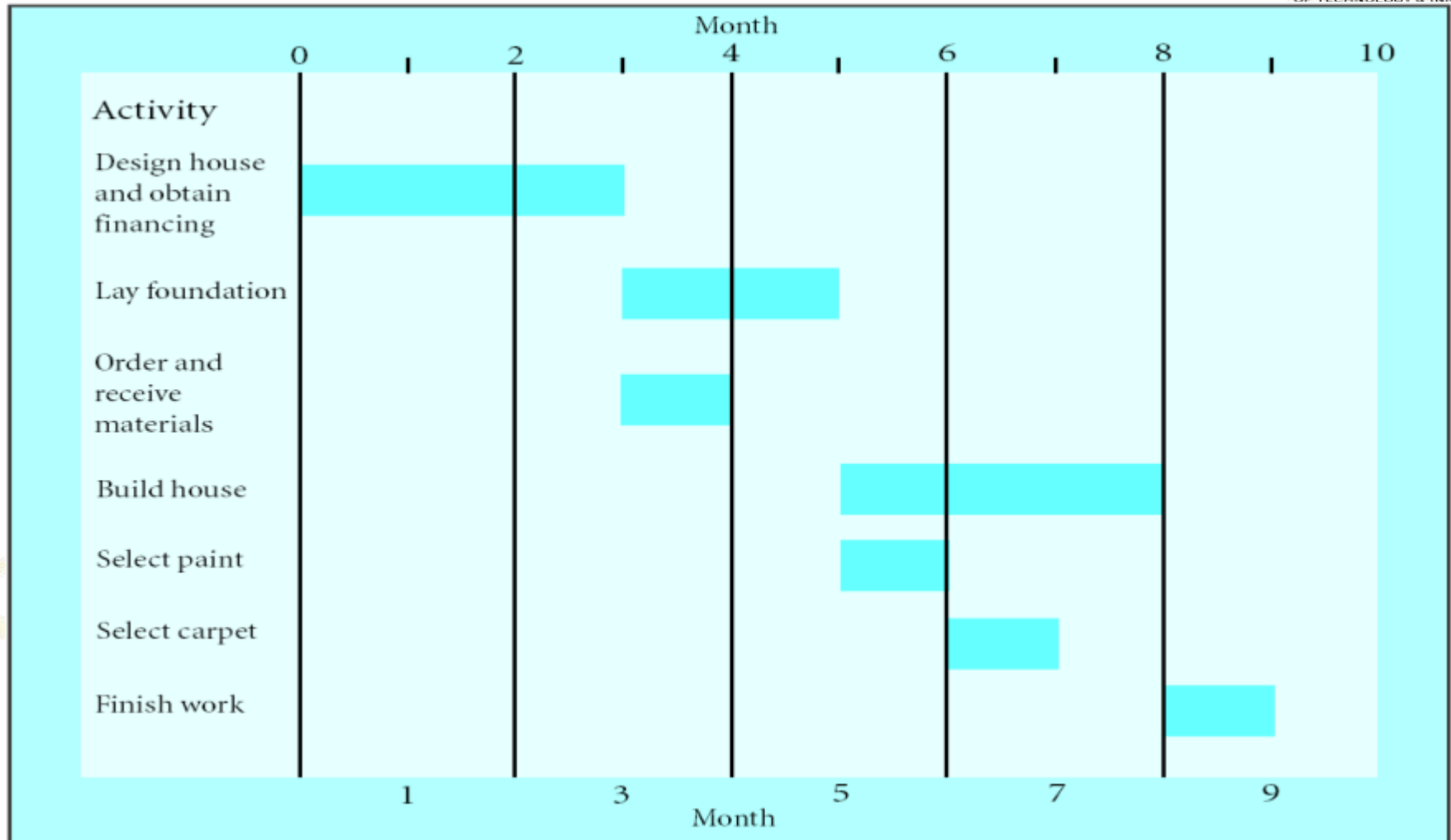
A Gantt chart

PERT/CPM

- PERT
  - Program Evaluation and Review Technique
  - Developed by U.S. Navy for Polaris missile project
  - Developed to handle uncertain activity times

- CPM
  - Critical Path Method
  - Developed by Du Pont & Remington Rand
  - Developed for industrial projects for which activity times generally were known

PERT/CPM

- PERT and CPM have been used to plan, schedule, and control a wide variety of projects:
  - R&D of new products and processes
  - Construction of buildings and highways
  - Maintenance of large and complex equipment
  - Design and installation of new systems
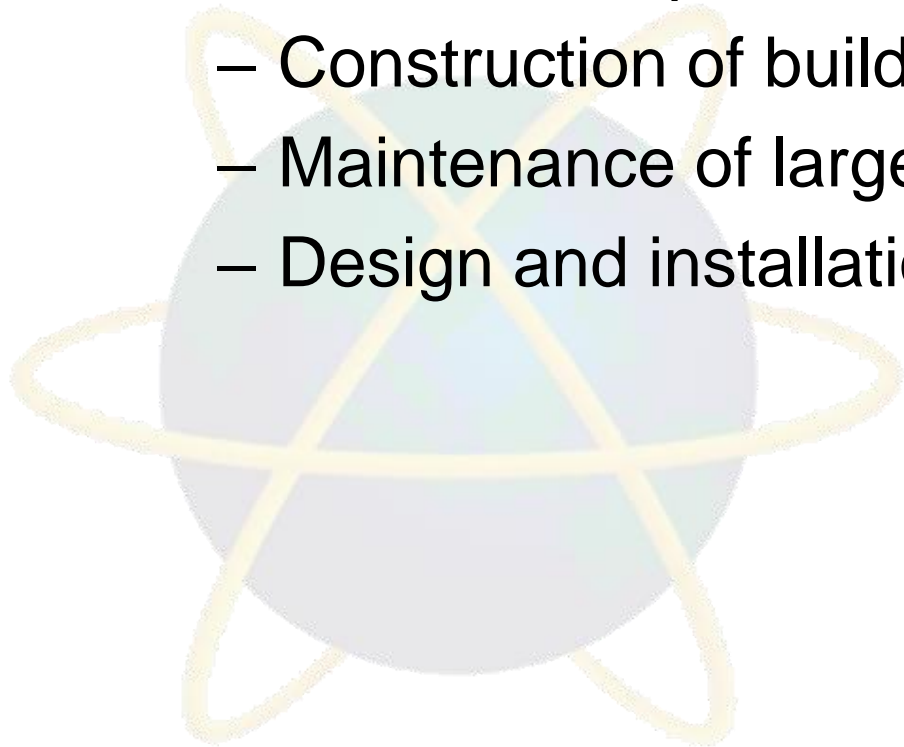
PERT/CPM

- PERT/CPM is used to plan the scheduling of individual <u>activities</u> that make up a project.

- Projects may have as many as several thousand activities.

- A complicating factor in carrying out the activities is that some activities depend on the completion of other activities before they can be started.

PERT/CPM

- Project managers rely on PERT/CPM to help them answer questions such as:

  - What is the <u>total time</u> to complete the project?

  - What are the <u>scheduled start and finish dates</u> for each specific activity?

  - Which activities are <u>critical</u> and must be completed exactly as scheduled to keep the project on schedule?

  - How long can <u>noncritical activities</u> be delayed before they cause an increase in the project completion time?

Example: Frank's Fine Floats

Frank's Fine Floats is in the business of building elaborate parade floats. Frank and his crew have a new float to build and want to use PERT/CPM to help them manage the project. The table on the next slide shows the activities that comprise the project. Each activity's estimated completion time (in days) and immediate predecessors are listed as well.

Frank wants to know the total time to complete the project, which activities are critical, and the earliest and latest start and finish dates for each activity.

Example: Frank's Fine Floats

Activity	Description	Immediate Predecessors	Completion Time (days)
A	Initial Paperwork	---	3
B	Build Body	A	3
C	Build Frame	A	2
D	Finish Body	B	3
E	Finish Frame	C	7
F	Final Paperwork	B,C	3
G	Mount Body to Frame	D,E	6
H	Install Skirt on Frame	C	2

Example: Frank's Fine Floats

- Project Network
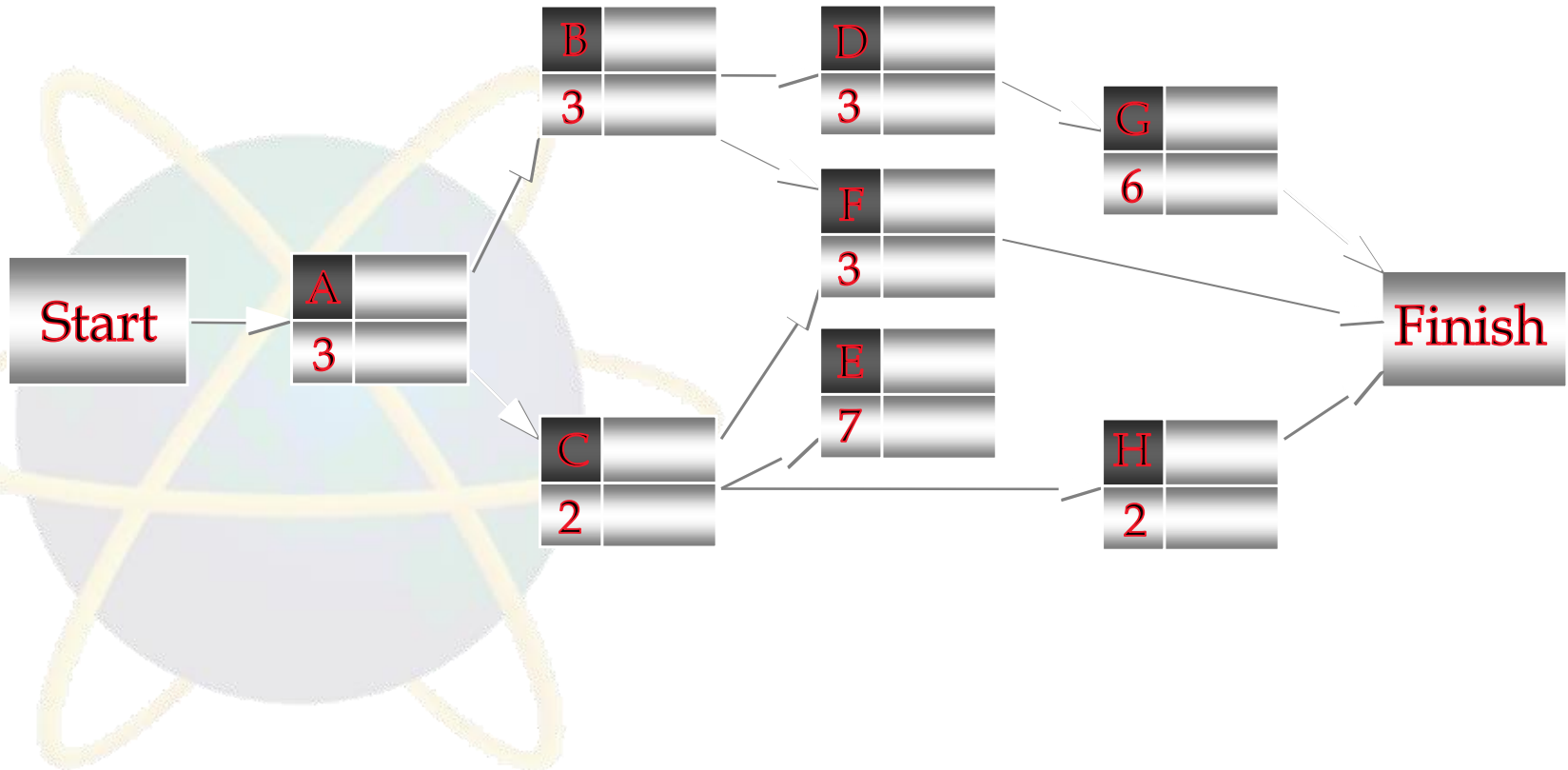
Earliest Start and Finish Times

- Step 1: Make a forward pass through the network as follows: For each activity i beginning at the Start node, compute:

  – <u>Earliest Start Time</u> = the maximum of the earliest finish times of all activities immediately preceding activity i. (This is 0 for an activity with no predecessors.)

  – <u>Earliest Finish Time</u> = (Earliest Start Time) + (Time to complete activity i).

  The project completion time is the maximum of the Earliest Finish Times at the Finish node.

Example: Frank's Fine Floats

- Earliest Start and Finish Times
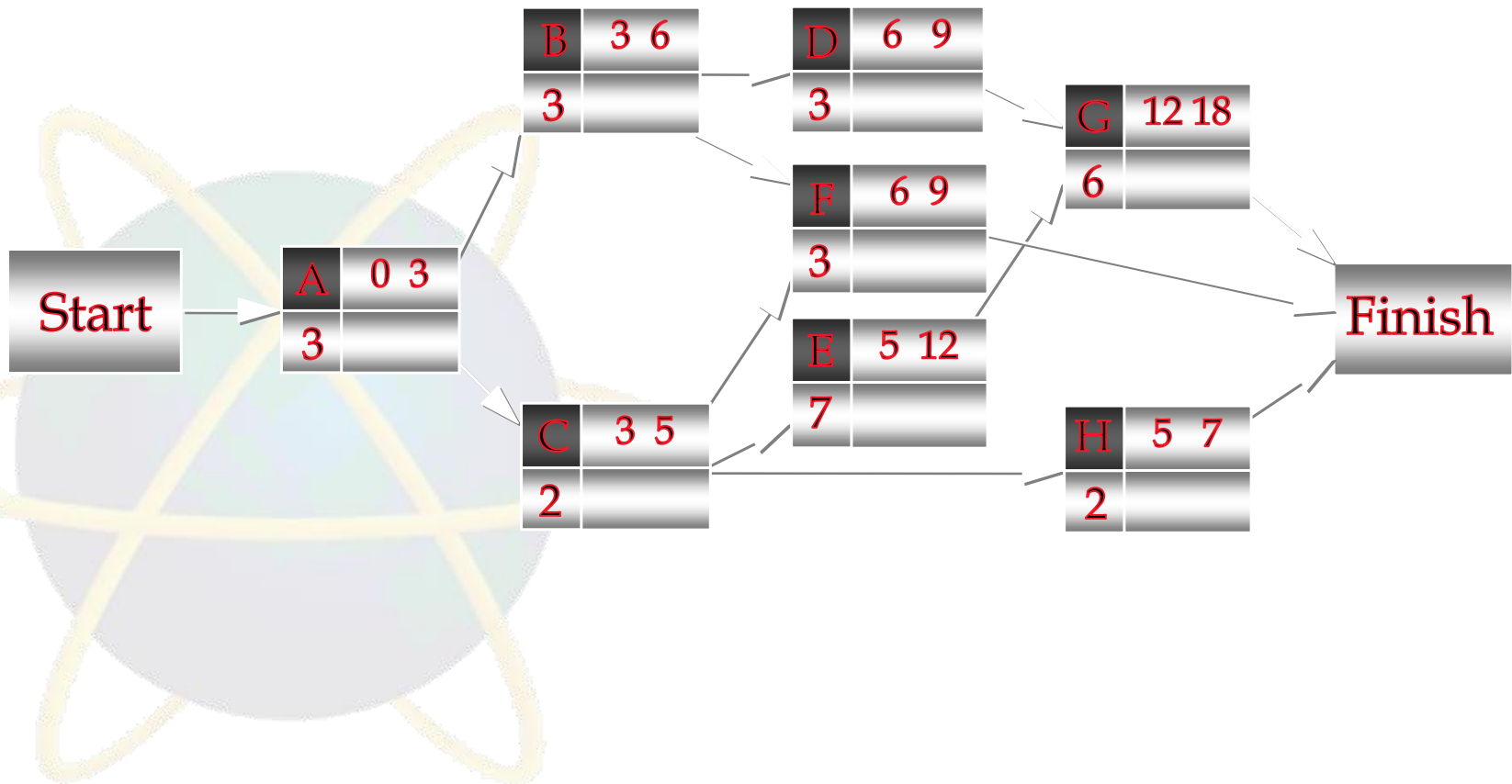
Latest Start and Finish Times

- Step 2: Make a backwards pass through the network as follows: Move sequentially backwards from the Finish node to the Start node. At a given node, j, consider all activities ending at node j. For each of these activities, i, compute:

  - Latest Finish Time = the minimum of the latest start times beginning at node j. (For node N, this is the project completion time.)

  - Latest Start Time = (Latest Finish Time) - (Time to complete activity i).

Example: Frank's Fine Floats

• Latest Start and Finish Times
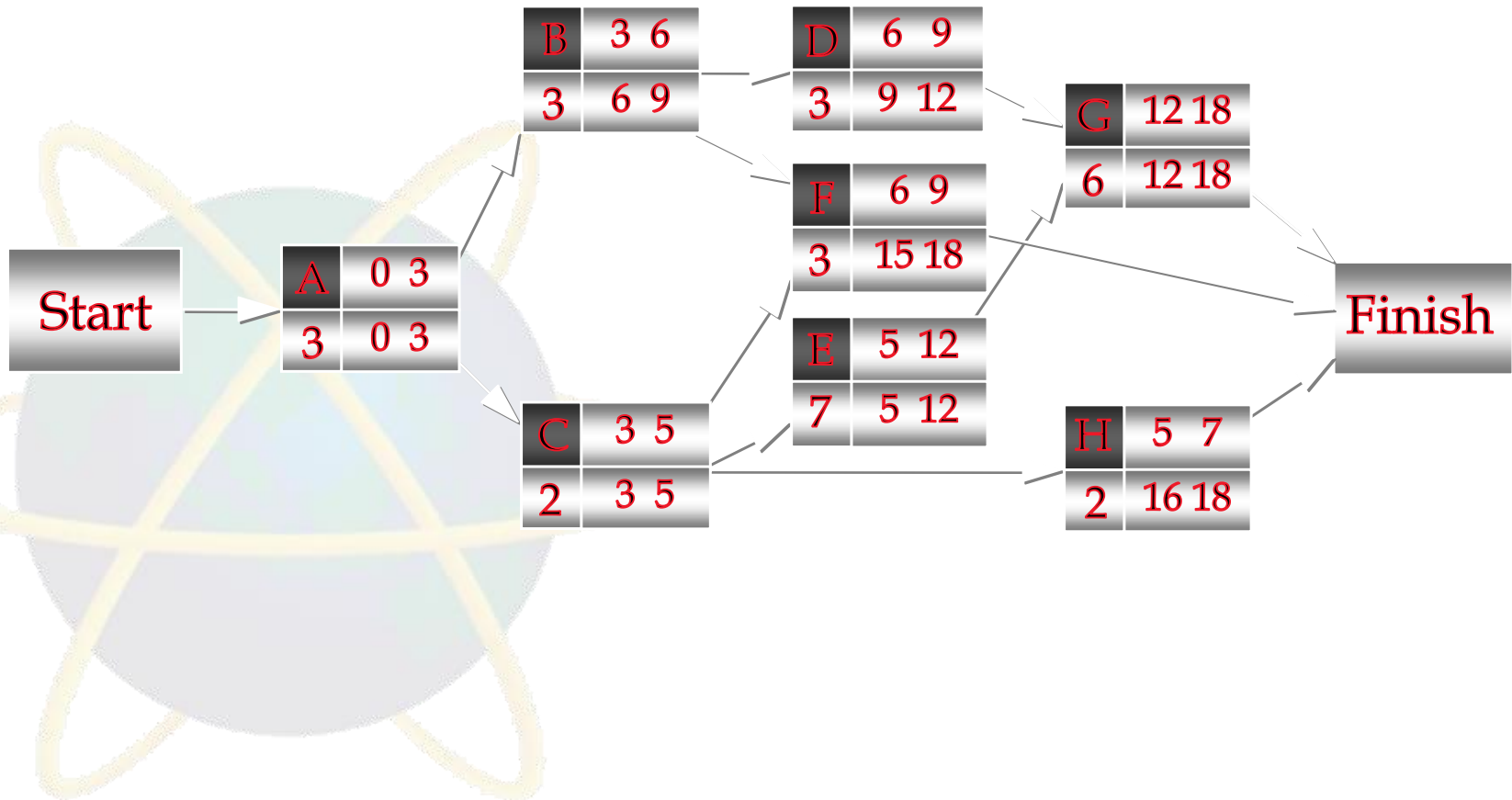
Determining the Critical Path

- Step 3:

Calculate the slack time for each activity by:

Slack = (Latest Start) - (Earliest Start), or

= (Latest Finish) - (Earliest Finish).
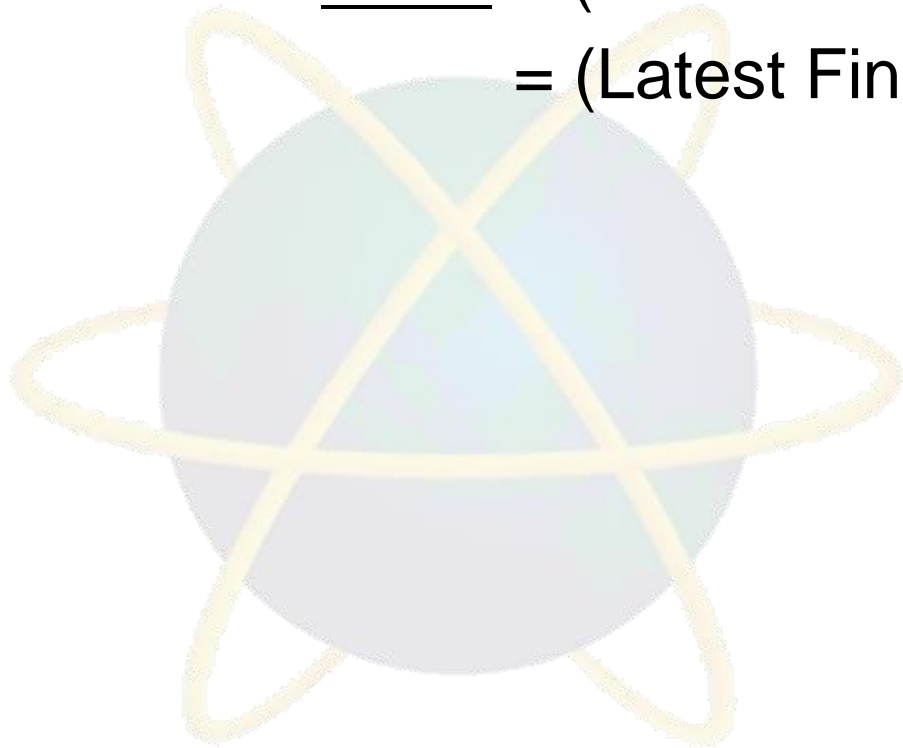
Example: Frank's Fine Floats

- Activity Slack Time

Activity	ES	EF	LS	LF	Slack	
A	0	3	0	3	0	(crit.)
B	3	6	6	9	3	
C	3	5	3	5	0	(crit.)
D	6	9	9	12	3	
E	5	12	5	12	0	(crit.)
F	6	9	15	18	9	
G	12	18	12	18	0	(crit.)
H	5	7	16	18	11	

Example: Frank's Fine Floats

- Determining the Critical Path

  – A <u>critical path</u> is a path of activities, from the Start node to the Finish node, with 0 slack times.

  – Critical Path: A – C – E – G

  – The <u>project completion time</u> equals the maximum of the activities' earliest finish times.

  – Project Completion Time: 18 days

Example: Frank's Fine Floats

- Critical Path
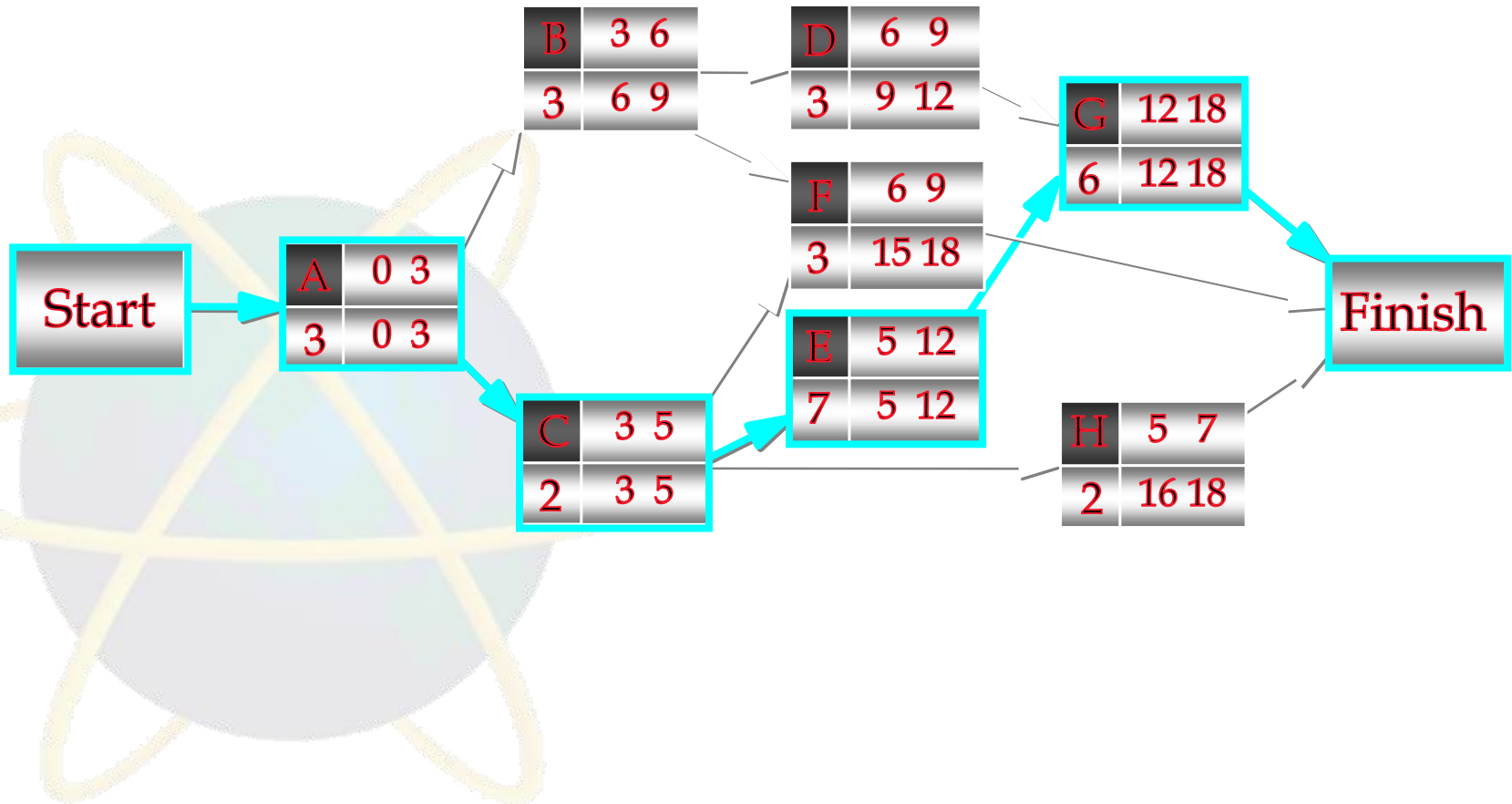
Example

Activity duration and its preceding activities are shown in the following table. Determine the critical path and slack.

		Immediate Predecessor	Activity Time (months)
A	Prepare drawing	-	5
B	Identify new tenants	-	6
C	Develop prospectus	A	4
D	Select contractor	A	3
E	Prepare permits	A	1
F	Obtain permits	E	4
G	Construction	D, F	14
H	Finalise tenant contracts	B, C	12
I	Tenants move in	G, H	2

Uncertain Activity Times

- In the <u>three-time estimate approach</u>, the time to complete an activity is assumed to follow a Beta distribution.

- An activity's <u>mean completion time</u> is:

$$t = (a + 4m + b)/6$$

- An activity's <u>completion time variance</u> is:

$$\sigma^2 = ((b\text{-}a)/6)^2$$

- a = the <u>optimistic</u> completion time estimate
- b = the <u>pessimistic</u> completion time estimate
- m = the <u>most likely</u> completion time estimate

Uncertain Activity Times

- In the three-time estimate approach, the critical path is determined as if the mean times for the activities were fixed times.

- The <u>overall project completion time</u> is assumed to have a normal distribution with mean equal to the sum of the means along the critical path and variance equal to the sum of the variances along the critical path.

Example: ABC Associates

- Consider the following project:

Activity	Immed. Predec.	Optimistic Time (Hr.)	Most Likely Time (Hr.)	Pessimistic Time (Hr.)
A	--	4	6	8
B	--	1	4.5	5
C	A	3	3	3
D	A	4	5	6
E	A	0.5	1	1.5
F	B,C	3	4	5
G	B,C	1	1.5	5
H	E,F	5	6	7
I	E,F	2	5	8
J	D,H	2.5	2.75	4.5
K	G,I	3	5	7

- Project Network
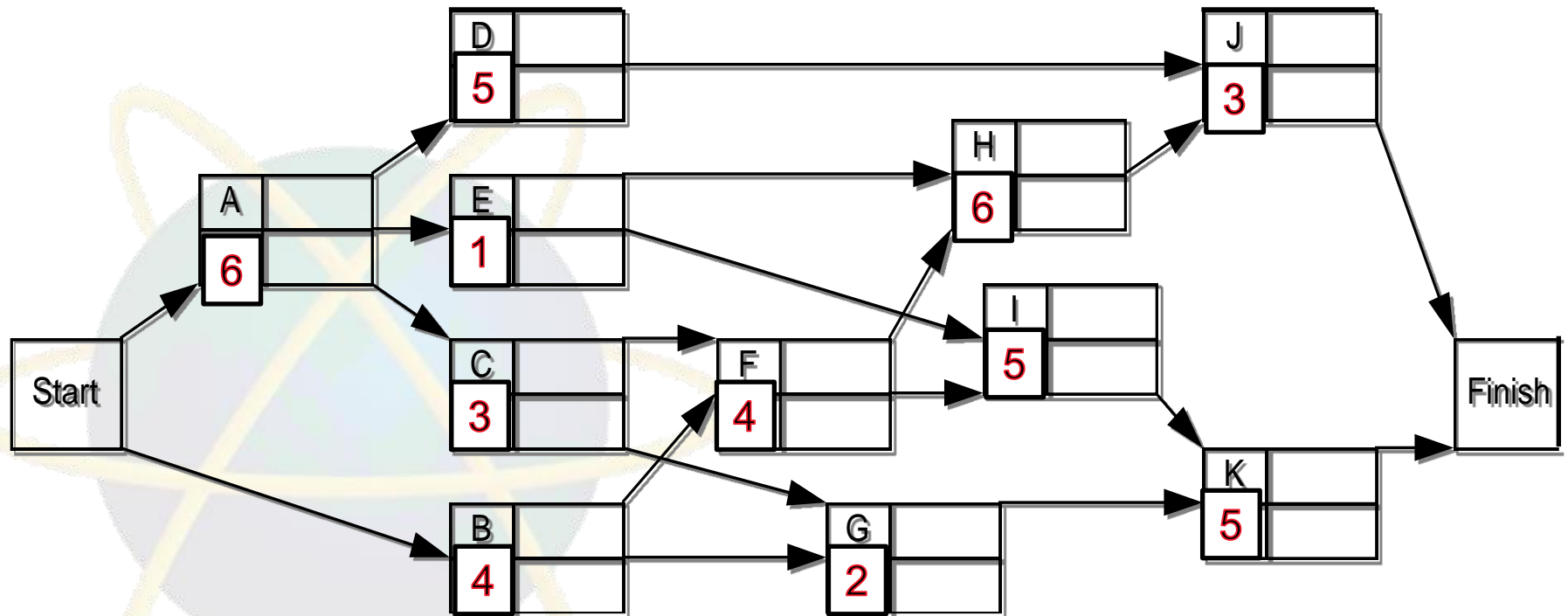
Example: ABC Associates

- Activity Expected Times and Variances

$$t = (a + 4m + b)/6 \qquad \sigma^2 = ((b-a)/6)^2$$

Activity	Expected Time	Variance
A	6	4/9
B	4	4/9
C	3	0
D	5	1/9
E	1	1/36
F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

Example: ABC Associates

- Earliest/Latest Times and Slack

Activity	ES	EF	LS	LF	Slack
A	0	6	0	6	0 *
B	0	4	5	9	5
C	6	9	6	9	0 *
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0 *
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0 *
J	19	22	20	23	1
K	18	23	18	23	0 *

Example: ABC Associates

- Determining the Critical Path
    - A <u>critical path</u> is a path of activities, from the Start node to the Finish node, with 0 slack times.
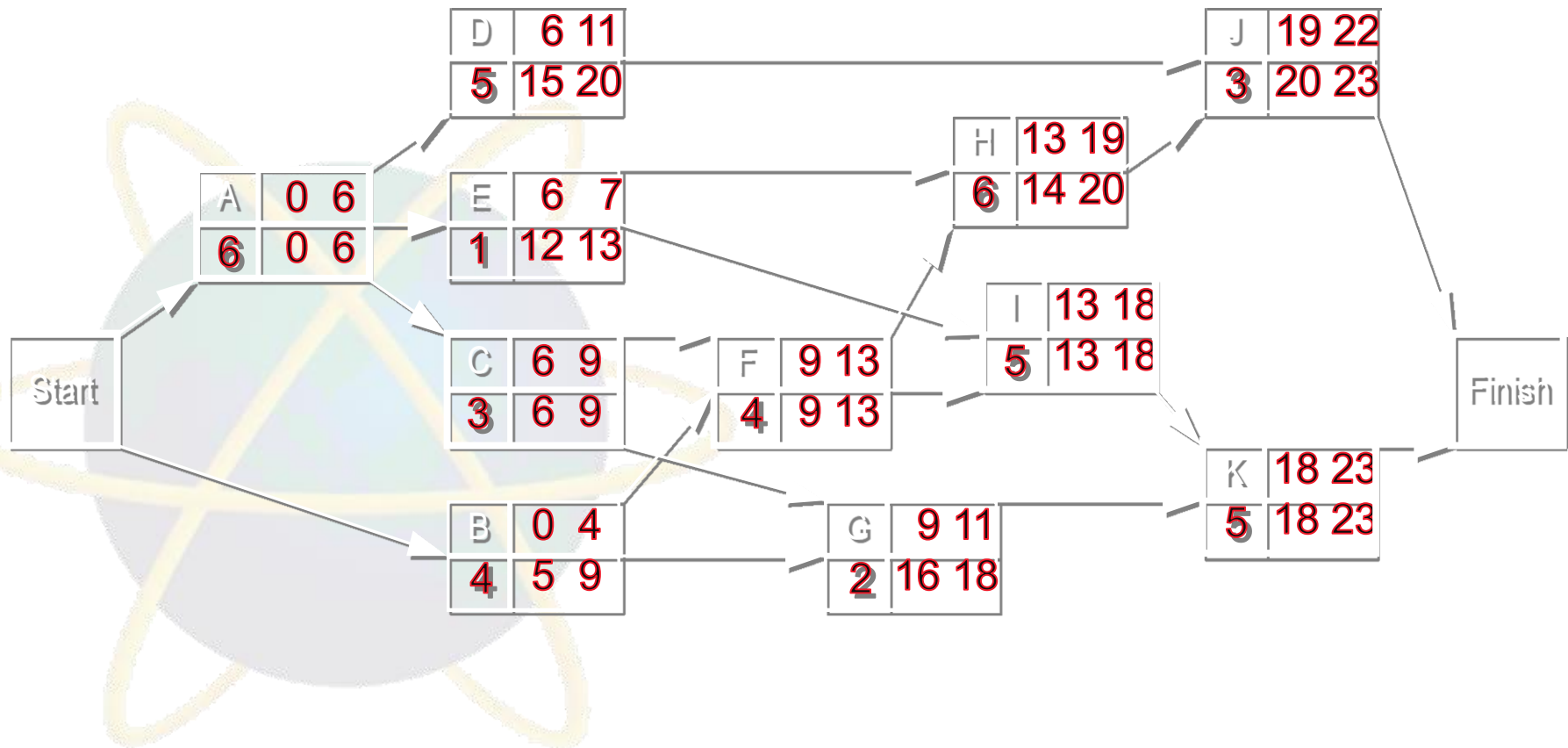
    - Critical Path: A – C – F – I – K

    - The <u>project completion time</u> equals the maximum of the activities' earliest finish times.

    - Project Completion Time: 23 hours

Example: ABC Associates

- Critical Path (A-C-F-I-K)

D	6 11
5	15 20

J	19 22
3	20 23

H	13 19
6	14 20

A	0 6
6	0 6

E	6 7
1	12 13

I	13 18
5	13 18

Start

C	6 9
3	6 9

F	9 13
4	9 13

Finish

K	18 23
5	18 23

B	0 4
4	5 9

G	9 11
2	16 18

Example: ABC Associates

- Probability the project will be completed within 24 hrs

$$\sigma^2 = \sigma^2_A + \sigma^2_C + \sigma^2_F + \sigma^2_I + \sigma^2_K$$

$$= 4/9 + 0 + 1/9 + 1 + 4/9$$

$$= 2$$

$$\sigma = 1.414$$

$$z = (24 - 23)/\sigma = (24-23)/1.414 = .71$$

From the Standard Normal Distribution table:

$$P(z \leq .71) = .5 + .2612 = \boxed{.7612}$$

Exercise

		Immediate Predecessor	Optimistic Time	Most Likely	Pessimistic Time
A	Prepare design	-	4	5	12
B	Plan market research	-	1	1.5	5
C	Routing	A	2	3	4
D	Prototype	A	3	4	11
E	Marketing brochure	A	2	3	4
F	Cost estimates	C	1.5	2	2.5
G	Testing	D	1.5	3	4.5
H	Market survey	B, E	2.5	3.5	7.5
I	Pricing and Forecast	H	1.5	2	2.5
J	Final report	F, G, I	1.0	2	3

Based on the following activity table, determine whether the project above can be completed within 20 week.

Quick Review Question
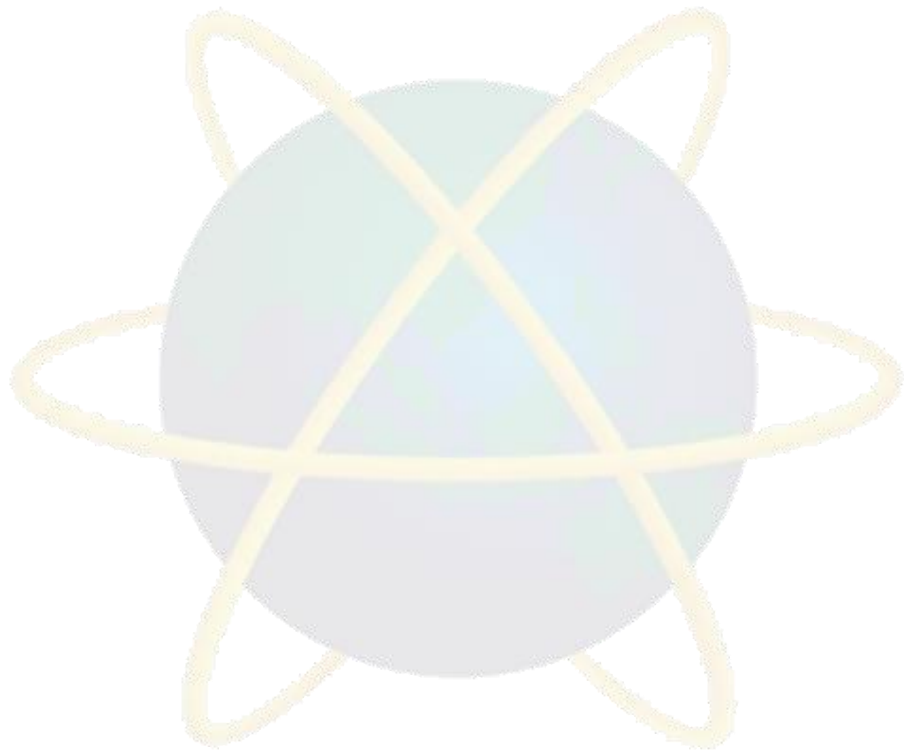
Follow Up Assignment
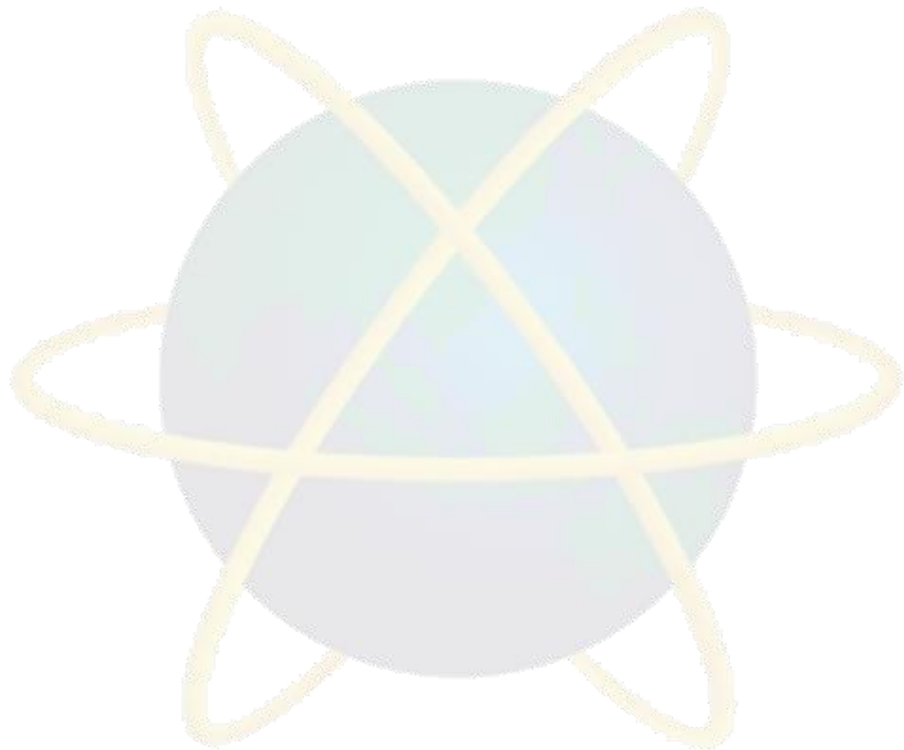
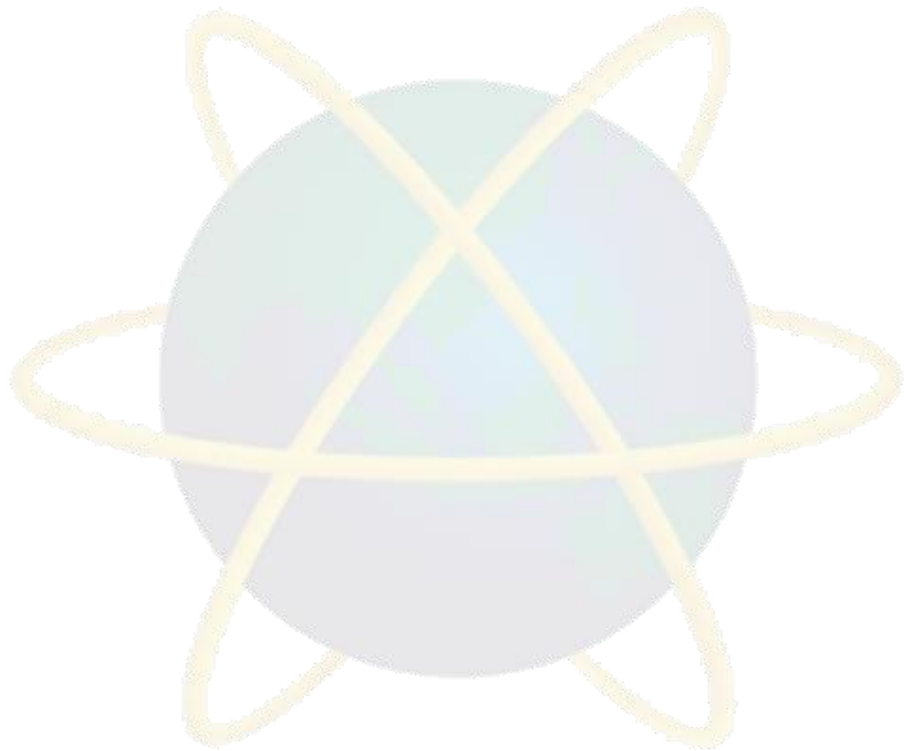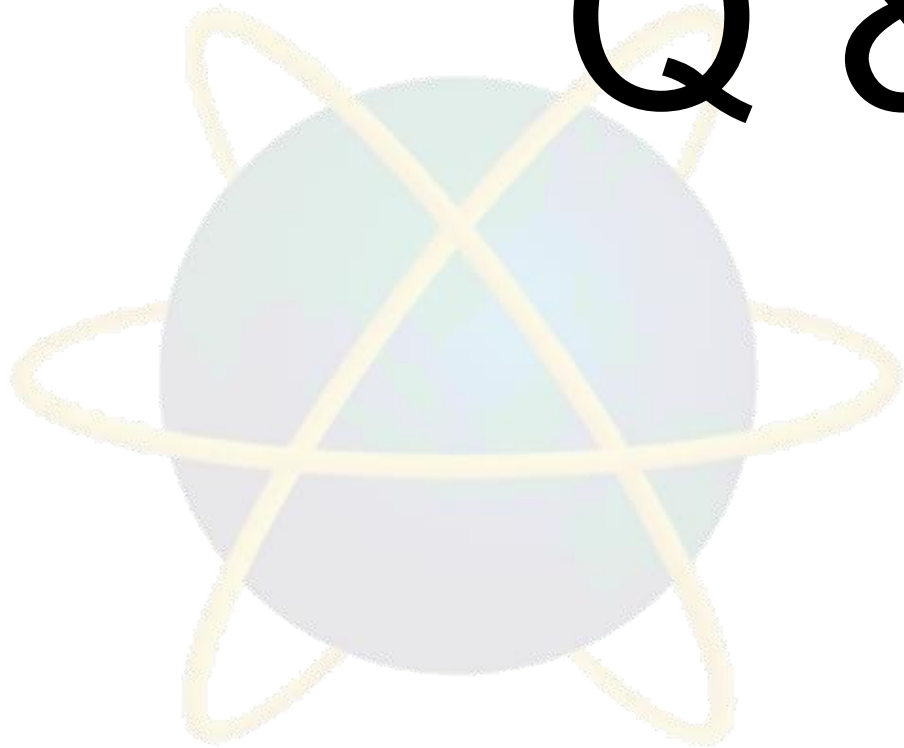# **Summary of Main Teaching Points**

Question and Answer Session

Q & A

Next Lesson

END